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Research Article

Exact Solutions and Dynamic Properties of the Perturbed Nonlinear Schrödinger Equation with Conformable Fractional Derivatives Arising in Nanooptical Fibers

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The main idea of this paper is to investigate the exact solutions and dynamic properties of a space-time fractional perturbed nonlinear Schrödinger equation involving Kerr law nonlinearity with conformable fractional derivatives. Firstly, by the complex fractional traveling wave transformation, the traveling wave system of the original equation is obtained, then a conserved quantity, namely, the Hamiltonian, is constructed, and the qualitative analysis of this system is conducted via this quantity by classifying the equilibrium points. Moreover, the existences of the soliton and periodic solution are established via the bifurcation method. Furthermore, all exact traveling wave solutions are constructed to illustrate our results explicitly by the complete discrimination system for the polynomial method.

1. Introduction

For hundred of years, the partial differential equation plays a vital role in many fields of science, and constructing exact solution to it could help us gain a deeper insight into the corresponding phenomena. However, traditional integer-order equation sometimes could not meet the requirement of modeling some special problems such as anomalous diffusion [1]; thus, the fractional calculus is proposed to handle this. Fractional calculus has many definitions, for example, the traditional Riemann–Liouville (RL) definition [2], modified RL definition [3], and conformal definition [4]. However, the classical RL definition is very complex to apply, and the modified RL definition has already been proved wrong [5, 6]; thus, choosing a proper fractional definition is an important and difficult task.

In this paper, we consider the following space-time fractional perturbed nonlinear Schrödinger equation (7).

$$iq_t^\alpha + aq_x^{2\alpha} + b|q|^2 q - i\left(\left(\sigma q_x^\alpha - \lambda |q|^2 q\right)_x^\alpha - \gamma \left|q^2\right|_x^\alpha q\right) = 0, \quad (1)$$

with conformable fractional derivatives, where α is the corresponding fractional order; $q(x, t)$ is the complex valued function defining wave profile in optical fibers; a and b represent the group velocity dispersion and nonlinear term, respectively; σ is the intermodal dispersion; λ is the self-steepeninng perturbation term; and γ is the nonlinear dispersion coefficient. The nonlinear Schrödinger equation has a broad applications in modeling light waves in nanooptical fibers [7–9]. Especially, when the external electric field exists, this equation can be used to solve nonharmonic motion of electrons bound in molecules [10], so constructing exact solutions to this equation is of great significance. In [11], some optical solitons and singular periodic wave solutions are constructed by the extended Sinh-Gördon equation expansion method, and W-shaped solitons are shown by Al-Ghafri and his colleagues [12]. Other results about soliton theory could be seen in [13–24].

The goal of this paper is to conduct qualitative and quantitative analysis to (1) by the complete discrimination system for polynomial method (CDSPM). The topological structure of this equation is shown, and the existences of soliton and periodic solution are also presented. Moreover, to verify our conclusion explicitly, all exact traveling wave solutions, namely, the classification of traveling wave solutions, are obtained. All conditions of parameters are discussed; thus, this paper contains all results of traveling wave solutions in the existing literatures, and some new solutions are obtained. To the best of our knowledge, this is the first time that the qualitative analysis is conducted to this equation, and we could also see the critical region of the existence of each kind of solution.

The CDSPM is proposed by Liu [25–27] and has been successfully applied to a series of integer-order [28, 29] and fractional-order equations [30–34]. Then, Kai et al. found that this method could also be used to conduct qualitative analysis [35], and combining with the bifurcation method, we can even establish the existence of the soliton and periodic solution [36–38].

The construction of this paper is as follows. The corresponding traveling wave system is given in Section 2, and qualitative analysis is conducted. Moreover, the existences of the soliton and periodic solution are also established in this section. To verify our conclusion explicitly, all single traveling wave solutions are constructed in Section 3, and concrete examples under concrete parameters are also shown to ensure the existence of each solution. In the final part, a brief discussion is given.

2. Dynamic Properties of Equation (1)

By setting

$$q(x,t)=u(\xi)e^{i(\phi(\xi)-wt)}, \tag{2}$$

where $\xi=(x^{\alpha}/\alpha)-(vt^{\alpha}/\alpha)$ [33], (1) becomes

$$-(v+\sigma)u'+2au'\phi'+au\phi''+(3\lambda+2\gamma)u^2u'=0, \tag{3}$$

for the real part, and

$$au''+wu+bu^3+(v+\sigma)u\phi'-au\left(\phi'\right)^2-\lambda u^3\phi'=0, \tag{4}$$

for the imaginary part. From (3), we have

$$\phi'=\frac{v+\sigma}{2a}-\frac{3\lambda+2\gamma}{4a}u^2-\frac{C}{a}u^{-2}. \tag{5}$$

Substituting (5) into (4) yields

$$u''=A_3u^5+A_2u^3+A_1u+A_0u^{-3}, \tag{6}$$

where $A_1=(C(\lambda+2\gamma-2aw+(v+\sigma)^2))/2a^2$, $A_2=(\lambda(v+\sigma)-2ab)/2a^2$, $A_3=(4\lambda\gamma+4\gamma^2-3\lambda^2)/16a^2$, $A_0=C^2/a^2$, and C is a constant of integration. By setting $u^2=V$, we have

$$V''=\frac{8A_3}{3}V^3+3A_2V^2+4A_1V+4A_0. \tag{7}$$

Multiplying (6) with V' and integrating it once, we have

$$\left(V'\right)^2=a_4V^4+a_3V^3+a_2V^2+a_1V+a_0, \tag{8}$$

where $a_4=2A_3/3$, $a_3=A_2$, $a_2=2A_1$, $a_1=4A_0$, and a_0 is a constant of integration. (6) is equivalent to the following dynamic system:

$$\begin{aligned}V'&=U,\\ U'&=\frac{8A_3}{3}V^3+3A_2V^2+4A_1V+4A_0,\end{aligned} \tag{9}$$

and thus, the corresponding Hamiltonian is given by

$$H(U,V)=U^2-\left(a_4V^4+a_3V^3+a_2V^2+a_1V\right). \tag{10}$$

Now, let us show that the Hamiltonian (12) is a conserved quantity. Taking derivative of right side of (12) with respect to ξ, we have

$$\begin{aligned}&2UU'-\left(4a_4V^3+3a_3V^2+2a_2V+a_1\right)V'\\ &\quad=2U\left[U'-\left(\frac{8A_3}{3}V^3+3A_2V^2+4A_1V+4A_0\right)\right]=0,\end{aligned} \tag{11}$$

which just proves our conclusion, and we can also conclude that the global phase portrait to the system (11) are just the contour lines of the Hamiltonian (12). In the following, we shall conduct qualitative analysis through this quantity by introducing the complete discrimination system.

From the Hamiltonian (12), we can see that the derivative of the potential energy is given by

$$U_1'(V)=-4a_4\left(V^3+b_2V^2+b_1V+b_0\right), \tag{12}$$

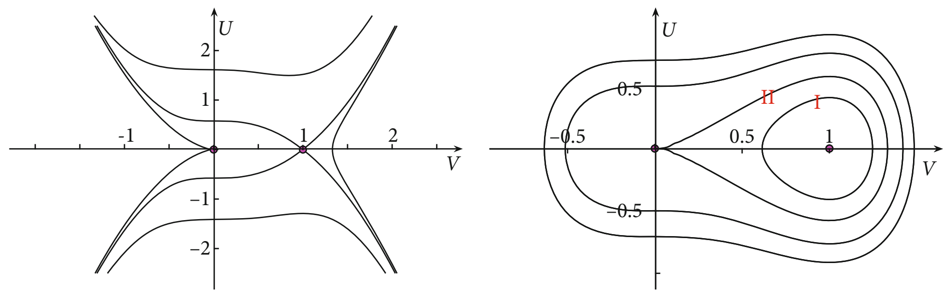
where $b_2=3a_3/4a_4$, $b_1=a_2/2a_4$, and $b_0=a_1/4a_4$. By introducing the following discrimination system

$$\begin{aligned}\Delta&=-27\left(\frac{2}{27}b_2^3+b_0-\frac{b_1b_2}{3}\right)^2-4\left(b_1-\frac{b_2^2}{3}\right)^3,\\ D&=b_1-\frac{1}{3}b_2^2,\end{aligned} \tag{13}$$

we can see that four cases need to be discussed here. This is just the general procedure of the CDSPM—first, rewriting the original equation into the integral form and then introducing the complete discrimination system to discussing the relation between the roots of the corresponding polynomial and the parameters. Then, we can find that all the conditions are discussed, and the results we obtained are integrated.

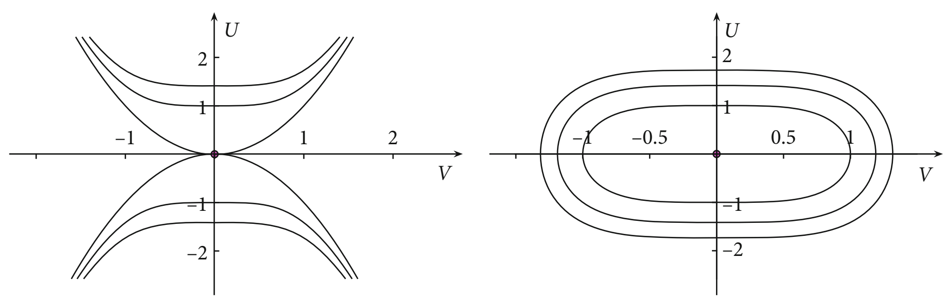
Case 1. $D<0$, $\Delta=0$, we can get

$$U'(V)=-4a_4(V-s)^2(V-l),\ (s\neq l). \tag{14}$$



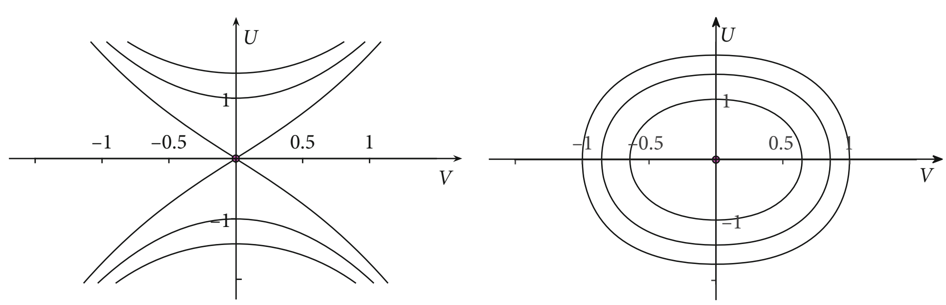
(a) (b)

Figure 1: The phase portraits of (8) in Case 1: (a) $a_4 = 1$ and (b) $a_4 = -1$.



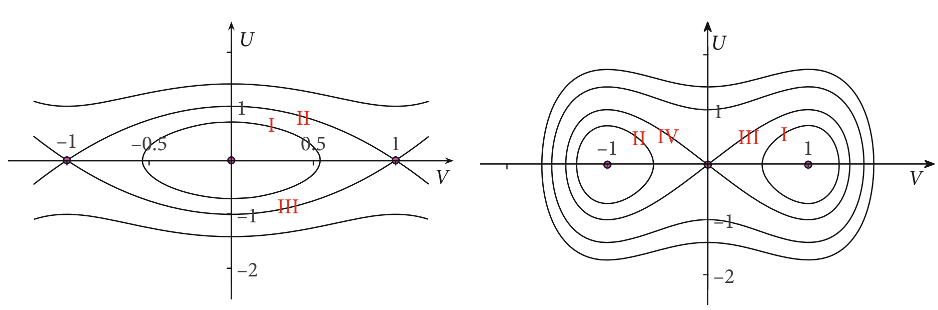
(a) (b)

Figure 2: The phase portraits of (8) in Case 2: (a) $a_4 = 1$ and (b) $a_4 = -1$.



(a) (b)

Figure 3: The phase portraits of (8) in Case 3: (a) $a_4 = 1$ and (b) $a_4 = -1$.



(a) (b)

Figure 4: The phase portraits of (8) in Case 4: (a) $a_4 = 1$ and (b) $a_4 = -1$.

$(s,0)$ is a cuspidal point and $(l,0)$ is a center for $a_4>0$ and a saddle point when $a_4<0$. For example, when $a_3=4/3$, $a_2=a_1=0$, we have $s=0, l=1$, and corresponding global phase portraits are shown in Figure 1 when $a_4=\pm 1$.

From Figure 1(b), we can see that trajectory I is a closed orbit with a center inside, which just indicates the existence of the periodic solution, and trajectory II is a homoclinic orbit, which indicates the existence of the bell-shaped soliton solution [36].

Case 2. $D=0$, $\Delta=0$, we have

$$U'(V) = -4a_4(V-s)^3. \tag{15}$$

There is only one equilibrium point $(s,0)$ here. It is a cuspidal point when $a_4>0$ and a center when $a_4<0$. For example, when $a_3=a_2=a_1=0$, we have $s=0$, and the corresponding global phase portraits can be seen in Figure 2 when $a_4=\pm 1$. From Figure 2(b), we can also conclude that the original equation has periodic solution.

Case 3. $\Delta<0$, we have

$$U_{1'}(V) = -4a_4(V-s)\left(V^2+sV+l\right), \left(s^2-4l<0\right). \tag{16}$$

There is also only one equilibrium point $(s,0)$ in this case. It is a saddle point when $a_4>0$ and a center when $a_4<0$. So, this case is very similar to Case 2. For example, when $a_3=a_1=0$ and $a_2=2$, we have $s=-1$, and the global phase portrait is given in Figure 3 when $a_4=\pm 1$.

Case 4. $D<0$, $\Delta>0$, we have

$$U_{1'}(V) = -4a_4(V-s)(V-l)(V-m). \tag{17}$$

This case is rather interesting due to that there are three equilibrium points $(s,0)$, $(l,0)$, and $(m,0)$ here. When $a_4>0$, $(s,0)$, and $(m,0)$ are two saddle points and $(l,0)$ is a center, and whereas for $a_4<0$, $(s,0)$, and $(m,0)$ are two centers and $(l,0)$ is a saddle point. Concrete examples of global phase portraits when $s=-m=1, l=0$, and $a_4=\pm 1$ are given in Figure 4.

For Figure 4(a), we can see that trajectory I is a closed orbit with a center inside, which indicates the existence of the periodic solution, and trajectory II and III are two heteroclinic orbits, which indicates the existence of the kink and antikink solitary wave solution, respectively. Figure 4(b) is a "figure eight loop" with trajectories I and II are two closed orbits with a center inside, which indicates the existence of the periodic solution, and trajectories III and IV are two homoclinic orbits, which means the corresponding equation has bright and dark bell-shaped soliton solution.

Now, we have showed the topological structure of system (3) and established the existences of the soliton and the periodic solution. In order to verify the conclusion explicitly, we construct the classification of traveling wave solutions to (3) by the CDSPM.

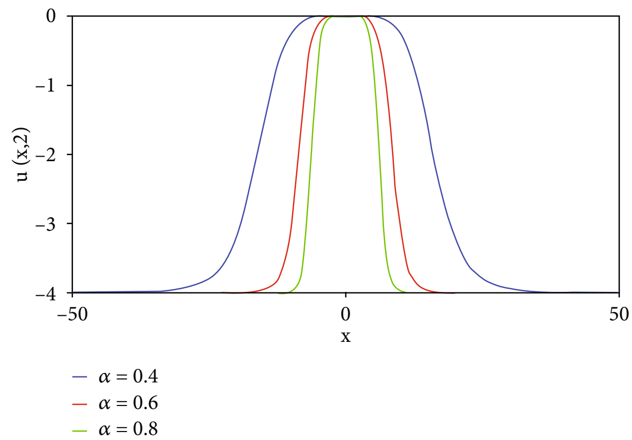


Figure 5: Figure of solution (31) with various fractional orders when $v=2$ and t is fixed at 2.

2.1. Traveling Wave Solutions to (8). In this section, we construct all traveling wave solutions, namely, the classification of traveling wave solutions to (7). We only focus on the condition of $a_4>0$, and $a_4<0$ could be treated similarly.

By taking the following transformation,

$$\varphi = (a_4)^{1/4}\left(u+\frac{a_3}{4a_4}\right), \xi_1 = (a_4)^{1/4}\xi, \tag{18}$$

(8) becomes

$$\left(\varphi_{\xi_1}'\right)^2 = \varphi^4 + p\varphi^2 + q\varphi + r = F(\varphi), \tag{19}$$

where $p=a_2/\sqrt{a_4}$, $q=((a_3^3/8a_2^4)-(a_2a_3/2a_4)+a_1)a^{-1/4}$ and $r=(-a_3^4/256a_4^3)+(a_2a_3^2/16a_4^2)-(a_1a_3/4a_4)+a_0$. First, we need to introducing the following complete discrimination system:

$$\begin{aligned}
&D_1 = 4, D_2 = -p, D_3 = -2p^3 + 8pr - 9q^2,\\
&D_4 = -p^2q^2 + 4p^4q + 36pq^2r - 32p^2q^2 - \frac{27}{4}q^4 + 64r^3,\\
&E_2 = 9p^2 - 32pr.
\end{aligned} \tag{20}$$

The complete discrimination system (27) given in Section 2 is the third-order form, and here is the fourth order. By discussing the relation between the parameters and the coefficients, we shall see that every condition of parameters is discussed; thus, what we have obtained is the classification of traveling wave solutions.

Case 1. $D_2<0, D_3=0$ and $D_4=0$, $F(\varphi)$ has a pair of double conjugate complex roots, namely,

$$F(\varphi) = \left(\varphi^2+s^2\right)^2, (s>0), \tag{21}$$

by substituting (35) into (7), we have the following solution:

$$\varphi = s\tan\left(s(\xi_1 - \xi_0)\right), \tag{22}$$

where ξ_0 is a constant of integration. (36) is a trigonometric function periodic solution. For example if $p = 4, q = 0$ and $r = 4$, then, we have $s = 2$ and the solution (36) is given by

$$\varphi = 2\tan 2(\xi_1 - \xi_0). \tag{23}$$

Case 2. When $D_2 = D_3 = D_4 = 0$. $F(\varphi)$ has a real root of multiplicities four, namely,

$$F(\varphi) = \varphi^4, \tag{24}$$

which leads to

$$\xi_1 - \xi_0 = \int \frac{d\varphi}{\varphi^2} = -\frac{1}{\varphi^{-1}}. \tag{25}$$

For example, when $p = q = r = 0$, we have

$$\varphi = -\frac{1}{\xi_1 - \xi_0}. \tag{26}$$

Case 3. When $D_2 > 0, D_3 = D_4 = 0$ and $E_2 > 0$, $F(\varphi)$ has two double distinct real roots, then we have

$$F(\varphi) = (\varphi - s)^2(\varphi - l)^2, (s > l), \tag{27}$$

which yields

$$\pm(\xi_1 - \xi_0) = \int \frac{d\varphi}{(\varphi - \mu)(\varphi - \nu)} = \frac{1}{\mu - \nu}\ \ln\left|\frac{\varphi - \mu}{\varphi - \nu}\right|. \tag{28}$$

If $\varphi > s$ or $\varphi < l$, we can get

$$\varphi = \frac{l - s}{2}\left[\coth\ \frac{\pm(s - l)(\xi_1 - \xi_0)}{2} - 1\right] + s, \tag{29}$$

and when $l < \varphi < s$, we have

$$\varphi = \frac{l - s}{2}\left[\tanh\ \frac{\pm(s - l)(\xi_1 - \xi_0)}{2} - 1\right] + l. \tag{30}$$

(30) is a solitary wave solution. (22) and (30) have verified the conclusion given in Section 2 that when $a_4 > 0$. (7) has periodic and soliton solution. This shows that the qualitative results obtained are truly correct.

For example, when $p = -8, r = 0$, and $q = 116$, we have $s = -l = 2$, then solitary wave solution (30) is given by

$$\varphi = -2[\tan h \pm (\xi_1 - \xi_0) - 1] - 2. \tag{31}$$

The corresponding figure of (31) is given in Figure 5. From it, we can see that the main impact of the fractional order is the velocity of convegence, and the position of the soliton is not influenced by it.

Case 4 $D_2 > 0, D_3 = D_4 = E_2 = 0$. $F(\varphi) = (\varphi - s)^3(\varphi - l)$, then the solution is given by

$$\pm(\xi_1 - \xi_0) = \int \frac{d\varphi}{(\varphi - s)\sqrt{(\varphi - s)(\varphi - l)}} = \frac{2}{l - s}\sqrt{\frac{\varphi - s}{\varphi - l}}. \tag{32}$$

Thus, the solution in explicit form is given by

$$\varphi = \frac{4(s - l)}{(l - s)^2(\xi_1 - \xi_0)^2 - 4} + s, \tag{33}$$

which is a rational solution. For example, when $p = -6, q = 8$, and $r = -3$, we have $l = 1, s = -3$, then

$$\varphi = \frac{4}{1 - 4(\xi_1 - \xi_0)^2} - 3. \tag{34}$$

Case 5. D 2D $3 < 0$, and $D_4 = 0$, we have

$$F(\varphi) = (\varphi - l)^2\left[(\varphi + l)^2 + s^2\right], \tag{35}$$

and then, we can get

$$\pm(\xi_1 - \xi_0) = \frac{1}{\sqrt{4l^2 + s^2}}\ \ln\left|\frac{\mu\varphi + \delta - \sqrt{(\varphi + l)^2 + s^2}}{\varphi - l}\right|, \tag{36}$$

where

$$\mu = \frac{3l}{\sqrt{4l^2 + s^2}}, \tag{37}$$

$$\delta = \sqrt{4l^2 + s^2} - \frac{3l^2}{\sqrt{4l^2 + s^2}}. \tag{38}$$

Thus,

$$\varphi = \frac{e^{\pm\sqrt{4l^2+s^2}(\xi_1-\xi_0)} - \mu + \sqrt{4l^2 + s^2}(2 - \mu)}{\left(e^{\pm\sqrt{4l^2+s^2}(\xi_1-\xi_0)} - \mu\right)^2 - 1}. \tag{39}$$

For example, when $p = -3, q = -2\sqrt{2}$, and $r = 6$, we have $l = \sqrt{2}, s = 1$, then the solution is given by

$$\varphi = \frac{e^{\pm 3(\xi_1-\xi_0)} - 4\sqrt{2} + 6}{\left(e^{\pm 3(\xi_1-\xi_0)} - \sqrt{2}\right)^2 - 1}. \tag{40}$$

Case 6. $D_i > 0 (i = 2, 3, 4)$. $F(\varphi)$ is given by

$$H(\varphi) = (\varphi - \alpha_1)(\varphi - \alpha_2)(\varphi - \alpha_3)(\varphi - \alpha_4), \tag{41}$$

where $\alpha_1 > \alpha_2 > \alpha_3 > \alpha_4$. Then, we can get the following elliptic function double periodic solutions. When $\alpha_4 > 0$, we have

$$\varphi=\frac{\alpha_2(\alpha_1-\alpha_4)sn^2\left(\left(\sqrt{(\alpha_1-\alpha_3)(\alpha_2-\alpha_4)}/2\right)(\xi_1-\xi_0),m\right)-\alpha_1(\alpha_2-\alpha_4)}{(\alpha_1-\alpha_4)sn^2\left(\left(\sqrt{(\alpha_1-\alpha_3)(\alpha_2-\alpha_4)}/2\right)(\xi_1-\xi_0),m\right)-(\alpha_2-\alpha_4)},\quad (\varphi>\alpha_1 or \varphi<\alpha_4), \tag{42}$$

$$\omega=\frac{\alpha_4(\alpha_2-\alpha_3)sn^2\left(\left(\sqrt{(\alpha_1-\alpha_3)(\alpha_2-\alpha_4)}/2\right)(\xi_1-\xi_0),m\right)-\alpha_3(\alpha_2-\alpha_4)}{(\alpha_2-\alpha_3)sn^2\left(\left(\sqrt{(\alpha_1-\alpha_3)(\alpha_2-\alpha_4)}/2\right)(\xi_1-\xi_0),m\right)-(\alpha_2-\alpha_4)},\quad (\alpha_3<\varphi<\alpha_2), \tag{43}$$

where $m^2=((\alpha_1-\alpha_4)(\alpha_2-\alpha_3))/((\alpha_1-\alpha_3)(\alpha_2-\alpha_4))$.

For $a_4<0$, similarly we can get

$$\varphi=\frac{\alpha_3(\alpha_1-\alpha_2)sn^2\left(\left(\sqrt{(\alpha_1-\alpha_3)(\alpha_2-\alpha_4)}/2\right)(\xi_1-\xi_0),m\right)-\alpha_2(\alpha_1-\alpha_3)}{(\alpha_1-\alpha_2)sn^2\left(\left(\sqrt{(\alpha_1-\alpha_3)(\alpha_2-\alpha_4)}/2\right)(\xi_1-\xi_0),m\right)-(\alpha_1-\alpha_3)},\quad (\alpha_1>\varphi>\alpha_2), \tag{44}$$

$$\varphi=\frac{\alpha_1(\alpha_3-\alpha_4)sn^2\left(\left(\sqrt{(\alpha_1-\alpha_3)(\alpha_2-\alpha_4)}/2\right)(\xi_1-\xi_0),m\right)-\alpha_4(\alpha_3-\alpha_1)}{(\alpha_3-\alpha_4)sn^2\left(\left(\sqrt{(\alpha_1-\alpha_3)(\alpha_2-\alpha_4)}/2\right)(\xi_1-\xi_0),m\right)-(\alpha_3-\alpha_1)},\quad (\alpha_4<\varphi<\alpha_3), \tag{45}$$

where $m^2=((\alpha_1-\alpha_2)(\alpha_3-\alpha_4))/((\alpha_1-\alpha_3)(\alpha_2-\alpha_4))$. For instance, when $p=-5, q=0, r=4$, and $\varphi>2$, we have $\alpha_1=2, \alpha_2=1, \alpha_3=-1, \alpha_4=-2$, and thus, the solution is given by

$$\varphi=\frac{4sn^2\left(\left(\sqrt{9}/2\right)(\xi_1-\xi_0),8/9\right)-6}{4sn^2\left(\left(\sqrt{9}/2\right)(\xi_1-\xi_0),8/9\right)-3}. \tag{46}$$

Case 7. $D_2\ D_3\ \geq 0$ and $D_4<0$. $F(\varphi)$ is given by

$$F(\varphi)=(\varphi-\mu)(\varphi-\nu)\left(\varphi-l^2\right)+s^2), \tag{47}$$

where $\mu>\nu$ and $s>0$. By setting

$$a=\frac{1}{2}(\mu+\nu)c-\frac{1}{2}(\mu-\nu)d, \tag{48}$$

$$b=\frac{1}{2}(\mu+\nu)d-\frac{1}{2}(\mu-\nu)c, \tag{49}$$

$$c=\mu-l-\frac{s}{m_1}, \tag{50}$$

$$d=\mu-l-sm_1, \tag{51}$$

$$E=\frac{s^2+(\mu-l)(\nu-l)}{s(\mu-\nu)}, \tag{52}$$

$$m_1=E+\sqrt{E^2+1}, \tag{53}$$

we can get

$$\varphi=\frac{acn\left(\left(\sqrt{\mp 2sm_1(\mu-\nu)}/2mm_1\right)(\xi_1-\xi_0),m\right)+b}{ccn\left(\left(\sqrt{\mp 2sm_1(\mu-\nu)}/2mm_1\right)(\xi_1-\xi_0),m\right)+d}, \tag{54}$$

where $m^2=1/(1+m_1^2)$. For instance, when $p=3$, $q=-4$, then $\mu=1$, $\nu=-1$ and $l=0$, $s=2$, the following solution could just be obtained

$$\varphi=-cn\left(\frac{\sqrt{\mp 2sm_1(\mu-\nu)}}{2mm_1}(\xi_1-\xi_0),m\right), \tag{55}$$

which is also an elliptic function double periodic solution.

Case 8. $D_2\ D_3\leq 0$ and $D_4>0$, we can get

$$F(\varphi)=\left((\varphi-l_1)^2+s_1^2\right)\left((\varphi-l_2)^2+s_2^2\right),(s_1\geq s_2>0). \tag{56}$$

By setting

$$a=l_1c+s_1d, \tag{57}$$

$$b=l_1d-s_1c, \tag{58}$$

$$c=-s_1-\frac{s_2}{m_1}, \tag{59}$$

$$d=l_1-l_2, \tag{60}$$

$$E=\frac{(l_1-l_2)^2+s_1^2+s_2^2}{2s_1s_2}, \tag{61}$$

$$m_1=E+\sqrt{E^2-1}, \tag{62}$$

we have

$$\varphi=\frac{asn(\eta(\xi_1-\xi_0),m)+bcn(\eta(\xi_1-\xi_0),m)}{csn(\eta(\xi_1-\xi_0),m)+dcn(\eta(\xi_1-\xi_0),m)}, \tag{63}$$

where $m^2=(m_1^2-1)/m_1^2$ and $\eta=s_2\sqrt{(c^2+d^2)(m_1^2c^2+d^2)}/(c^2+d^2)$. For example, when $p=5$, $q=4$, we have $l_1=l_2=0$, $s_1=1, s_2=2$, then

$$\varphi=\frac{cn(4(\xi_1-\xi_0),3/4)}{sn(4(\xi_1-\xi_0),3/4)}. \tag{64}$$

Case 9. D_2 , D $3>0$, and $D_4=0$. $F(\varphi)$ is given by

$$F(\varphi)=(\varphi-s)(\varphi-l)(\varphi-m)^2,(s>l). \tag{65}$$

By setting

$$c=\frac{\alpha_1-\alpha_2}{2}\left(\frac{\alpha_1+\alpha_2}{2}-\alpha_3\right), \tag{66}$$

we have

$$\pm(\xi_1 - \xi_0) = \int \frac{d\varphi}{(\varphi - \alpha_3)\sqrt{\varphi - \alpha_1)(\varphi - \alpha_2)}}, \tag{67}$$

whose solution is given by

$$(\xi_1 - \xi_0) = -\frac{1}{\sqrt{c^2 - 1}} \ln \left|\frac{y - c_1}{y + c_1}\right|, \left(c^2 - 1 > 0\right), \tag{68}$$

$$(\xi_1 - \xi_0) = -\sqrt{(1 - c^2)} \arctan \frac{c+1}{1-c} y, \left(c^2 - 1 < 0\right), \tag{69}$$

where $c_1 = \sqrt{(c+1)/(c-1)}$ and $y = \sqrt{1 - ((\alpha_1 - \alpha_2)/ (\varphi - ((\alpha_1 + \alpha_2)\alpha_1 + \alpha_2/2)\alpha_1 + \alpha_2/2) + (\alpha_1 - \alpha_2)\alpha_1 - \alpha_2/2)}$, which is a triangular function periodic solution. For example, when $p = -1, q = 0$, we have $\alpha_1 = 1, \alpha_2 = -1$, and $\alpha_3 = 0$, which leads to

$$\varphi = \frac{1}{\cos 2(\xi_1 - \xi_0)}, \tag{70}$$

which is also a periodic solution.

3. Conclusion

In this paper, we consider a space–time fractional perturbed nonlinear Schrödinger equation arising from nanooptical fibers. By taking the complex fractional traveling wave transformation, the traveling wave system of the original equation is obtained, then the corresponding Hamiltonian is constructed, and the qualitative analysis is conducted by introducing the complete discrimination system. The topological structure is given, and the existences of the soliton and periodic solution are established via the bifurcation method. To verify our conclusion explicitly, every kind of traveling wave solutions is constructed by the CDSPM, and some of them are new. In order to analyze the influence of the fractional parameter, a concrete example of the soliton solution is given. From it, we can see to directly that how the fractional order impact the position of the soliton and how these solutions convergent to the same value when the dependent variable ξ tends to infinity. All of the results given in the present paper show the powerfulness of the method adopted in the paper. In the future, we shall further analyze the nonlinear Schrödinger equation with RL definition to give more results to this equation, and we would also like to promote this method to other nonlinear equations like the coupled Boussinesq equation.

Data Availability

All data generated or analyzed during this study are included in this article.

Conflicts of Interest

The authors declare that they have no conflict of interest.

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