Hindawi
Advances in Mathematical Physics
Volume 2022, Article ID 7192231, 18 pages
https://doi.org/10.1155/2022/7192231

Research Article

Analytical Solution of One-Dimensional Nonlinear Conformable Fractional Telegraph Equation by Reduced Differential Transform Method

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Received 31 January 2022; Revised 10 May 2022; Accepted 1 July 2022; Published 21 July 2022

Academic Editor: Andr Nicolet

In this paper, the reduced differential transform method (RDTM) is successfully implemented to obtain the analytical solution of the space-time conformable fractional telegraph equation subject to the appropriate initial conditions. The fractional-order derivative will be in the conformable (CFD) sense. Some properties which help us to solve the governing problem using the suggested approach are proven. The proposed method yields an approximate solution in the form of an infinite series that converges to a closed-form solution, which is in many cases the exact solution. This method has the advantage of producing an analytical solution by only using the appropriate initial conditions without requiring any discretization, transformation, or restrictive assumptions. Four test modeling problems from mathematical physics, conformable fractional telegraph equations in which we already knew their exact solution using other numerical methods, were taken to show the liability, accuracy, convergence, and efficiency of the proposed method, and the solution behavior of each illustrative example is presented using tabulated numerical values and two- and three-dimensional graphs. The results show that the RDTM gave solutions that coincide with the exact solutions and the numerical solutions that are available in the literature. Also, the obtained results reveal that the introduced method is easily applicable and it saves a lot of computational work in solving conformable fractional telegraph equations, and it may also find wide application in other complicated fractional partial differential equations that originate in the areas of engineering and science.

1. Introduction

Differential equations have made significant development in recent years, with applications in physics, chemical and industrial mathematics, processing and control theory, and fluid dynamics. The expansion of differentiation and integration to noninteger orders is known as fractional calculus. The subject has gained special importance in the last two or three decades [1–3]. Many phenomena in engineering and other sciences have been characterized mathematically by fractional derivatives [4–7]. These representations have offered good results in the modeling of real-world problems [8–10]. Recently, investigators have paid attention to studying the solution of partial differential equations of fractional order [11–13]. For instance, Singh [14] studied the new aspects of the blood alcohol model associated with the Hilfer fractional operator. Using this model, the solutions for the concentration of alcohol in the stomach and the concentration of alcohol in the blood have been obtained in the form of the extended Mittag-Leffler function by applying the Sumudu transform method. The authors of the publication [15] propose the q-homotopy analysis Elzaki transform technique (q-HAETM) to solve fractional multidimensional diffusion equations that represent density dynamics in a material undergoing diffusion. The authors used the Caputo–Fabrizio sense of fractional derivative throughout. In the year 2021, [16] employed an iterative scheme to obtain the solution of the rumor-spreading dynamical model in a social network associated with the Atangana-Baleanu derivative of noninteger order.

However, till now, not a single definition of the fractional-order derivative has been found [17]. Some of the known fractional derivatives are Riemann–Liouville, modified Riemann–Liouville, Caputo, Caputo–Fabrizio, Hadmard, Erdélyi–Kober, Riesz, Grunwald–Letnikov, Marchaud, and others [18]. Other definitions have also been provided by Kilbas et al. and Miller and Ross in [19, 20]. All known fractional derivatives satisfy one of the well-known properties of classical derivatives, namely, the linear property. However, the other properties of classical derivatives, such as the derivatives of a constant, are zero; the product rule, quotient rule, and chain rule either do not hold or are too complicated for many fractional derivatives. For instance, $D_a^{\alpha}(1)=0$ does not fulfill the Riemann-Liouville definition. In Caputo's definition, $f(x)$ is assumed to be differentiable; otherwise, one cannot use such a definition. Moreover, Liouville's theorem in the fractional setting does not hold. Therefore, it is clear that all definitions of fractional derivatives seem deficient regarding certain mathematical properties, such as Rolle's theorem and the mean value theorem [2].

To overcome these complications, Khalil et al. [21] lately proposed a well-extended definition of the noninteger order derivative in 2014, named conformable fractional derivative (CFD). This definition is formulated as follows:

Definition 1 (see [21]). For the initial real value a, the conformable fractional derivative $D_a^{\alpha}f(x)$ of a real function $f:[a,\infty)\longrightarrow \mathbb{R}$, $\alpha\in(0,1]$ is defined as

$$D_a^{\alpha}f(x)=\lim_{h\longrightarrow 0}\frac{f\left(x+h(x-a)^{1-\alpha}\right)-f(x)}{h},\quad \text{for all } x>a,\ \alpha\in(0,1]. \tag{1}$$

The initial value a can be zero, and if the limit exists, $f(x)$ is said to be partially α-differentiable at $t>0$.

The CFD Definition 1 is very similar to the classical derivative. It depends upon the basic limit definition and, consequently, allows the easy extension of some typical theorems in calculus that the existing definitions of fractional derivatives did not allow, due to its simple nature. Along with the CFD's Definition 1, various classical properties, such as the mean value theorem and the product, quotient, and chain rules, are fulfilled. Moreover, this definition is provided with the Leibniz rule, which other fractional derivatives cannot achieve (see [22]). Another study [23] conducted by Abdeljawad presented left and right conformable fractional derivatives and fractional integrals of higher-order concepts. In addition, he defined the fractional chain rule, fractional integration by part formulae, Gronwall inequality, fractional power series expansion, and fractional Laplace transform. Abul-Ez et al. [24] introduced a comprehensive study on the conformable fractional Legendre polynomials. They presented the shifted conformable fractional Legendre polynomials and described an applicable scheme using the collocation method to solve some fractional differential equations (FDEs) in the sense of conformable derivatives. Growing attention has been paid to exploring the conformable derivatives due to the enormous number of their meaningful applications in many fields of science. The authors of the papers [25, 26] used the methods such as the Adomian decomposition method, modified homotopy perturbation method, and variational iteration method in the conformable fractional derivative senses to obtain the approximate-analytical solution of some initial boundary value problems (IBVP) and one-dimensional cable differential equation (FCE). Recently, in [27], Rabha et al. introduced different vitalizations of the growth of COVID-19 by using controller terms based on the concept of conformable calculus. The authors of the paper [28] present an exhaustive study on the conformable fractional Gauss hypergeometric function (CFGHF). They start by solving the conformable fractional Gauss hypergeometric differential equation (CFGHDE) about the fractional regular singular points $x=1$ and $x=\infty$. Next, they established various generating functions of the CFGHF by developing some differential forms for the CFGHF. Subsequently, they introduce the conformable fractional integral representation and the fractional Laplace transform of CFGHF. As an application, and after making a suitable change of the independent variable, the authors provide general solutions of some known conformable fractional differential equations, which could be written by means of the CFGHF.

Through the paper [29], fractional Laplace transforms in terms of conformable fractional-order Bessel functions (CFBFs) are presented. Several important formulas of the fractional Laplace integral operator acting on the CFBFs are established and the solutions of a generalized class of fractional kinetic equations associated with the CFBFs in view of the fractional Laplace transform. Furthermore, an orthogonality relation of the CFBFs which is used to study an expansion of any analytic functions by means of CFBFs is derived. Ghanbari et al. [30] studied the dynamic behavior of allelopathic stimulatory phytoplankton species with Mittag-Leffler (ML) law by using the Atangana-Baleanu fractional derivative (ABC) introduced in [31]. They developed the proposed model by involving a new nonsingular and nonlocal fractional derivative with the ML kernel. Furthermore, the authors used the fixed-point theory to prove the existence and uniqueness of the solution of the fractional-order model of allelopathic stimulatory phytoplankton species with the Mittag-Leffler Law. In addition to this, there is a lot of work related to the ongoing advancement of fractional derivatives. As a result, monographs [32–34] are available to interested readers for extra information.

As the greatest importance of FDEs to their applicability in different fields of sciences, their solution represents nowadays a vigorous research area for scientists, and finding approximate and exact solutions to FPDEs is an important task [18]. However, PDEs, particularly nonlinear PDEs, are notoriously difficult to solve. To tackle this problem, researchers worked on finding adequate methods for those FPDEs. One of the greatest methods found by researchers is the reduced differential transform method (RDTM). What distinguishes this method is that it provides us with analytical approximations, which in many cases are exact solutions, in a rapidly convergent power series form with elegantly computed terms ([12, 35] and see the references therein).

Moreover, RDTM reduces the size of the calculations, and it solves the equations straightforwardly and directly without using Adomian's polynomial, perturbation, discretization, linearization or any other transformation, and restrictive conditions [35–37]. As a result, the RDTM can overcome the aforementioned constraints and restrictions of perturbation approaches as well as high computing complexity, allowing us to accurately evaluate nonlinear equations. Recently, the RDTM was successfully applied to different classes of differential equations. For instance, the authors of the paper [38] used this method to obtain the solutions of linear and nonlinear, ordinary, partial, fractional, and systems of differential equations. Also, the authors study the sufficient condition for convergence of the RDTM for nonlinear differential equations. In the paper [39], the authors used the RDTM defined with a conformable operator for solving time-fractional one-dimensional cable differential equation (FCE) modeling neuronal dynamics. Paper [40] presents the RDTM for finding the exact and approximate analytical solution for the conformable time-fractional Swift-Hohenberg (S-H) equations with and without dispersive terms. For extra information, see [12, 35, 41–43] and the references therein.

The telegraph equation is one of the classes of PDEs which expresses the distance and time on an electric transmission line with voltage and current. This equation is frequently used in the study of electric communications, wave propagation in wave phenomena, and cable transmission systems. The telegraph equation outperforms the heat equation in predicting parabolic physical events, and it has numerous applications in sectors including radio frequency, wireless signals, telephone lines, and microwave transmission [44]. Many practical implementations entail substituting a fractional derivative for the time derivative in the telegraph equation. The authors of the works [45–48] have investigated the time-fractional telegraph equation with Caputo fractional derivative intensively.

The conformable telegraph equations have a wide variety of applications in science and engineering, ideally in optimizing propagation-oriented and propagating electrical communication systems [48]. More recently, many researchers have given their attention to examining the solutions of conformable fractional telegraph equations using different methods. In 2021, [49] obtained accurate and convergent numerical solutions of linear space-time matching telegraph fractional equations employing a double conformable Sumudu matching transformation method. Moreover, the numerical model is equipped to explain the work, the accuracy of the work, and sobriety in its presentation method, and as a result, the proposed method shows an effective and convenient way to employ proven problems in science and engineering.

In paper [50], the authors used the conformable double Laplace transform and conformable double convolution, to solve a novel case of fractional telegraph equations with nonconstant coefficients by exchanging the nonhomogeneous terms by conformable double convolution functions. Bouaouid et al. [51] proved the existence, uniqueness, and stability of the integral solution of a nonlocal telegraph equation in the conformable time-fractional derivative using the cosine family of linear operators. The implicit fundamental solution is also given in terms of classical trigonometric functions. The Fourier approach for solving a conformable fractional telegraph equation is effectively implemented by the authors in [52]. They also go over the analytical solution of the conformable fractional telegraph equation with a nonhomogeneous Dirichlet boundary condition and show how to get it. However, the RDTM in the sense of conformable fractional derivative has not previously been studied to solve this problem.

The main goal of this paper is to apply the novel method RDTM in the sense of a conformable fractional derivative to obtain the analytical solution of the one-dimensional space-time fractional telegraph equation since most of the research existing in the literature focused on the numerical solutions for this problem. Using the described method, the solution to the current problem is obtained in the form of a convergent power series form. Then, the eighth-term, approximate solution of the present method is compared with the exact solution for each considered illustrative example in Tables 1–4 and 2D graphs 5, 6, 7, and 8. Additionally, the numerical simulation of the present method is also depicted by 3D Figures 1, 2, 3, and 4.

In this work, we are concerned with solving the following general conformable fractional one-dimensional telegraph equation given by

$$\begin{aligned}\frac{\partial^{2\alpha}u(x,t)}{\partial t^{2\alpha}}+a\frac{\partial^{\alpha}u(x,t)}{\partial t^{\alpha}}+bN(u(x,t))=c\frac{\partial^{2\beta}u(x,t)}{\partial x^{2\beta}}\\ +f(x,t),\quad x,t>0,\&\alpha,\beta\in(0,1],\end{aligned}\tag{2}$$

subject to the initial conditions

$$u(x,0)=g_0(x),\tag{3}$$

$$u_t(x,0)=g_1(x),\tag{4}$$

where a, b, and c are known real constants, N is the general linear and nonlinear operator, and $f(x,t)$ is the source term.

The remainder of this work is arranged in the following manner. The definitions, properties, and theorems of conformable fractional derivatives are discussed in Section 2. Section 3 presents some basic definitions, properties, and proofs of theorems related to the RDTM. Section 4 illustrates the description of the model; how the approximate analytical solution of the given telegraph equations is obtained using RDTM. In Section 5, we demonstrate the proposed method's reliability, convergence, and efficiency using four illustrative instances. Analytical answers and numerical simulations are presented in tables and graphs in Sections 5.1 and 5.2, respectively. In Section 6, we discuss the numerical results and illustrate the accuracy and efficiency of the RDTM. Finally, concluding remarks are outlined in Section 7.

2. Basic Definitions and Preliminaries of Conformable Fractional Derivative

Several definitions have been made to define the fractional derivative and continue to be done. In this section, we first present the two most commonly used definitions of

Table 1: Eighth-order approximate solution by RDTM of Example 1 for different values of α, β, and t and comparison with the exact solution.

α	β	x	t	Exact	RDTM (u_8)	Error of RDTM $\lvert u_8 - exact \rvert$
			0.01	2.7456	2.7456	7.4984×10^{-24}
			0.02	2.7732	2.7732	3.8430×10^{-21}
1	1	1	0.03	2.8011	2.8011	1.4789×10^{-19}
			0.04	2.8292	2.8292	1.9716×10^{-18}
			0.05	2.8577	2.8577	1.4704×10^{-17}
			0.01	9.0250	9.0250	1.0638×10^{-11}
			0.02	9.8045	9.8045	1.9746×10^{-10}
0.5	0.5	1	0.03	10.4480	10.4480	1.4755×10^{-9}
			0.04	11.0232	11.0232	5.5594×10^{-9}
			0.05	11.5561	11.5561	1.5250×10^{-8}

Table 2: Eighth-order approximate solution by RDTM of Example 2 for different values of α, β, and t and comparison with the exact solution.

α	β	x	t	Exact	RDTM (u_8)	Error of RDTM $\lvert u_8 - exact \rvert$
			0.01	2.6645	2.6645	3.8277×10^{-21}
			0.02	2.6117	2.6117	1.9559×10^{-18}
1	1	1	0.03	2.5600	2.5600	7.5040×10^{-17}
			0.04	2.5093	2.5093	9.9742×10^{-16}
			0.05	2.4596	2.4596	7.4166×10^{-15}
			0.01	4.9530	4.9530	5.1318×10^{-9}
			0.02	4.1968	4.1968	1.1428×10^{-7}
0.5	0.5	1	0.03	3.6957	3.6957	7.0009×10^{-7}
			0.04	3.32012	3.32012	2.5292×10^{-6}
			0.05	3.0210	3.0210	6.8431×10^{-6}

Table 3: Eighth-order approximate solution by RDTM of Example 3 for different values of α, β, and t and comparison with the exact solution.

α	β	x	t	Exact	RDTM (u_8)	Error of RDTM $\lvert u_8 - exact \rvert$
			0.01	0.0173	0.0173	4.8046×10^{-26}
			0.02	0.0171	0.0171	2.4575×10^{-23}
1	1	1	0.03	0.0170	0.0170	9.4381×10^{-22}
			0.04	0.0168	0.0168	1.2557×10^{-20}
			0.05	0.0166	0.0166	9.3466×10^{-20}
			0.01	0.0286	0.0286	4.8274×10^{-14}
			0.02	0.0263	0.0263	9.6199×10^{-13}
0.5	0.5	1	0.03	0.0247	0.0247	6.5831×10^{-12}
			0.04	0.0234	0.0234	2.4238×10^{-11}
			0.05	0.0223	0.0223	6.5860×10^{-11}

Table 4: Eighth-order approximate solution by RDTM of Example 19 for different values of α, β, and t and comparison with the exact solution.

α	β	x	t	Exact	RDTM (u_8)	Error of RDTM $\lvert u_8 - exact \rvert$
			0.01	0.0175	0.0175	8.7235×10^{-7}
			0.02	0.0175	0.0175	3.4893×10^{-6}
1	1	1	0.03	0.0175	0.0175	7.8506×10^{-6}
			0.04	0.0175	0.0175	3.5767×10^{-5}
			0.05	0.0175	0.0175	2.1804×10^{-5}
			0.01	0.0349	0.0349	6.9545×10^{-4}
			0.02	0.0349	0.0349	1.3862×10^{-3}
0.5	0.5	1	0.03	0.0349	0.0349	2.0725×10^{-3}
			0.04	0.0349	0.0349	2.7703×10^{-3}
			0.05	0.0349	0.0349	3.4311×10^{-3}

fractional derivatives, the Riemann-Liouville and Caputo definitions given in reference [7].

Definition 2. Riemann-Liouville's fractional derivative of a suitable mapping f is given as

$$D_x^{\alpha} f(x) = \frac{1}{\Gamma(n-\alpha)} \left(\frac{d}{dx}\right)^n \int_0^x (x-t)^{n-\alpha-1} f(t)dt, \quad n-1 < \alpha \leq n, \tag{5}$$

where α is the order of fractional derivative and $\alpha \in Z^+$.

Definition 3. Caputo fractional derivative of a suitable mapping f is given as

$$D_x^{\alpha} f(x) = \frac{1}{\Gamma(n-\alpha)} \int_0^x (x-t)^{n-\alpha-1} f^{(n)}(t)dt, \quad n-1 < \alpha \leq n, \tag{6}$$

where α is the order of fractional derivative and $\alpha \in Z^+$.

Definition 4. (see [31]). Consider that $f \in H^1(a,b)$, $b > a$, $\mu \epsilon [0,1]$ and differentiable; then, the Atangana-Baleanu arbitrary order derivatives in Caputo (ABC) and Riemann-

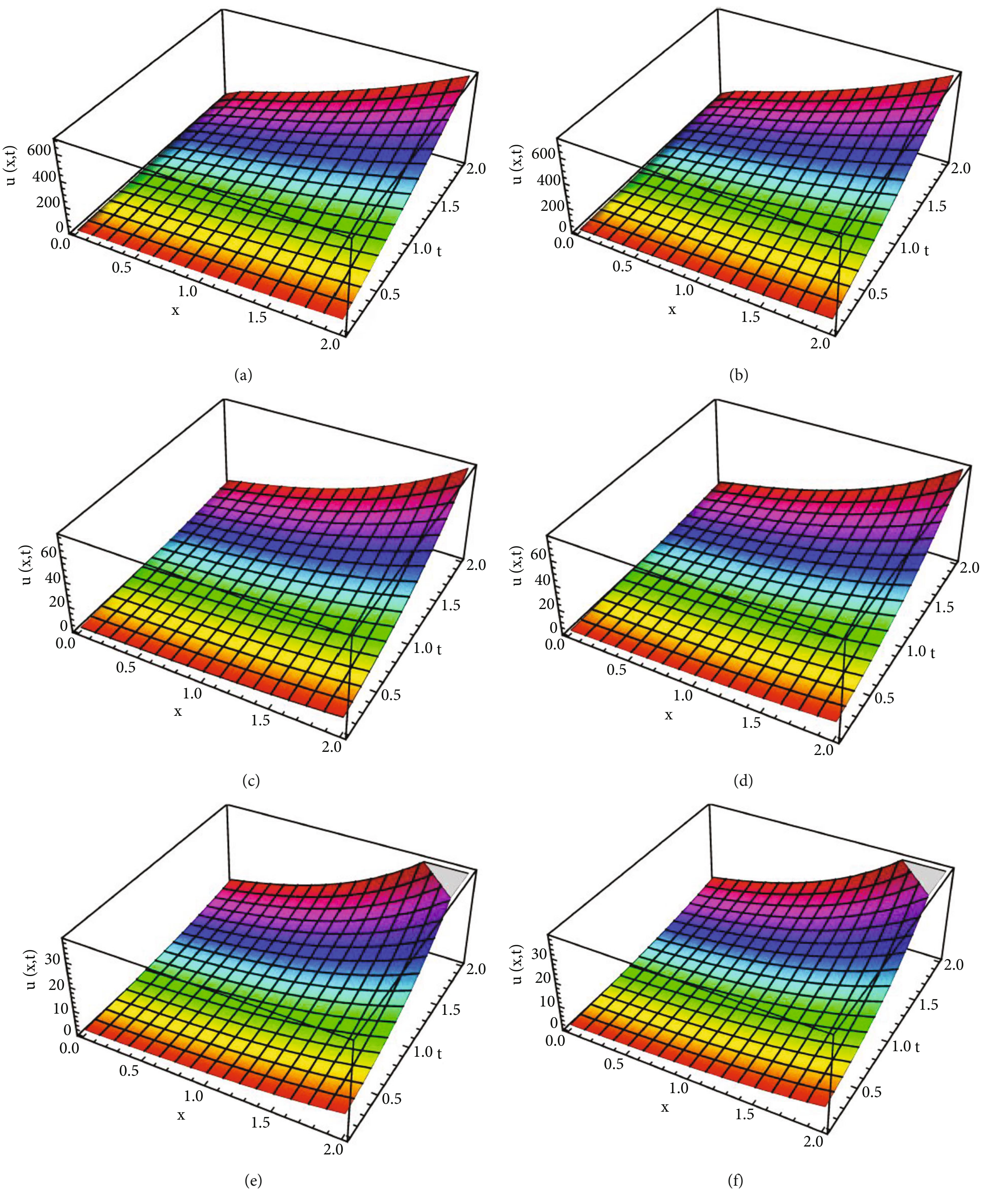


Figure 1: 3D solution plots of Example 1 obtained by the present method (a, c, e) in comparison with the exact solutions (b, d, f) at $\alpha = \beta = 0.4$, 0.8, and 1.

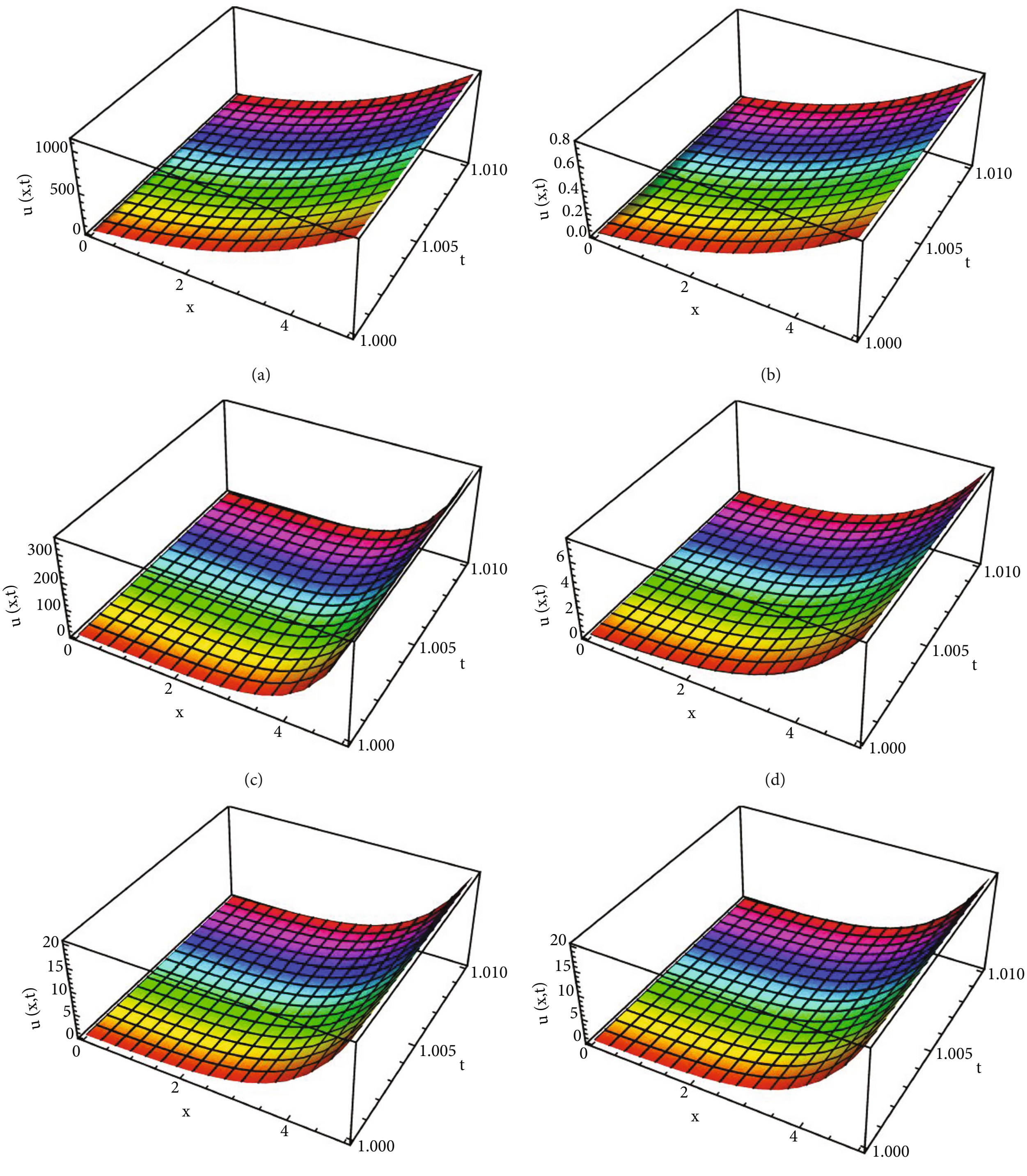


FIGURE 2: 3D solution plots of Example 2 obtained by the present method (a, c, e) in comparison with the exact solutions (b, d, f) at $\alpha = \beta = 0.4$, 0.8, and 1.

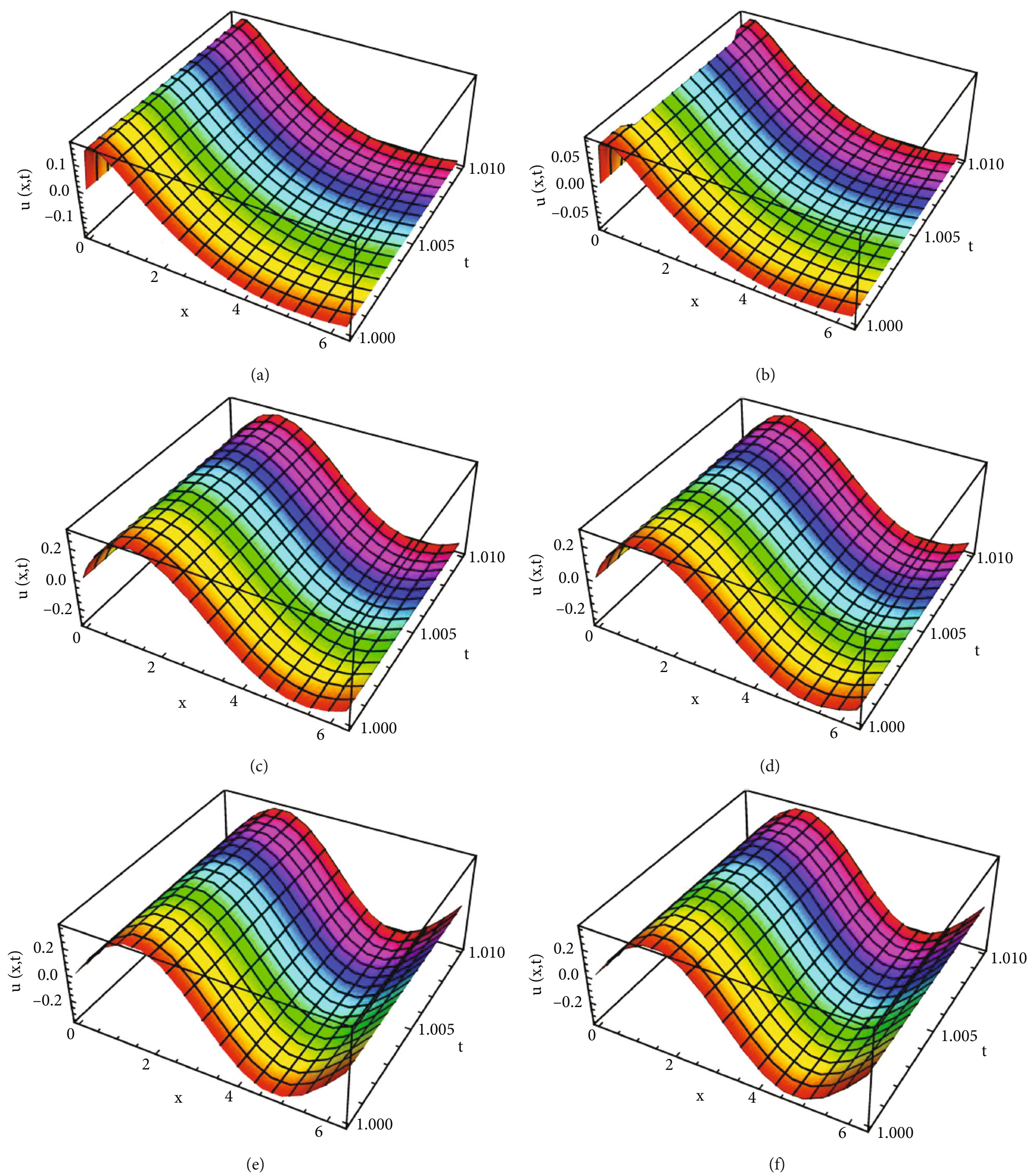


Figure 3: 3D solution plots of Example 3 obtained by the present method (a, c, e) in comparison with the exact solutions (b, d, f) at $\alpha = \beta = 0.4$, 0.8, and 1.

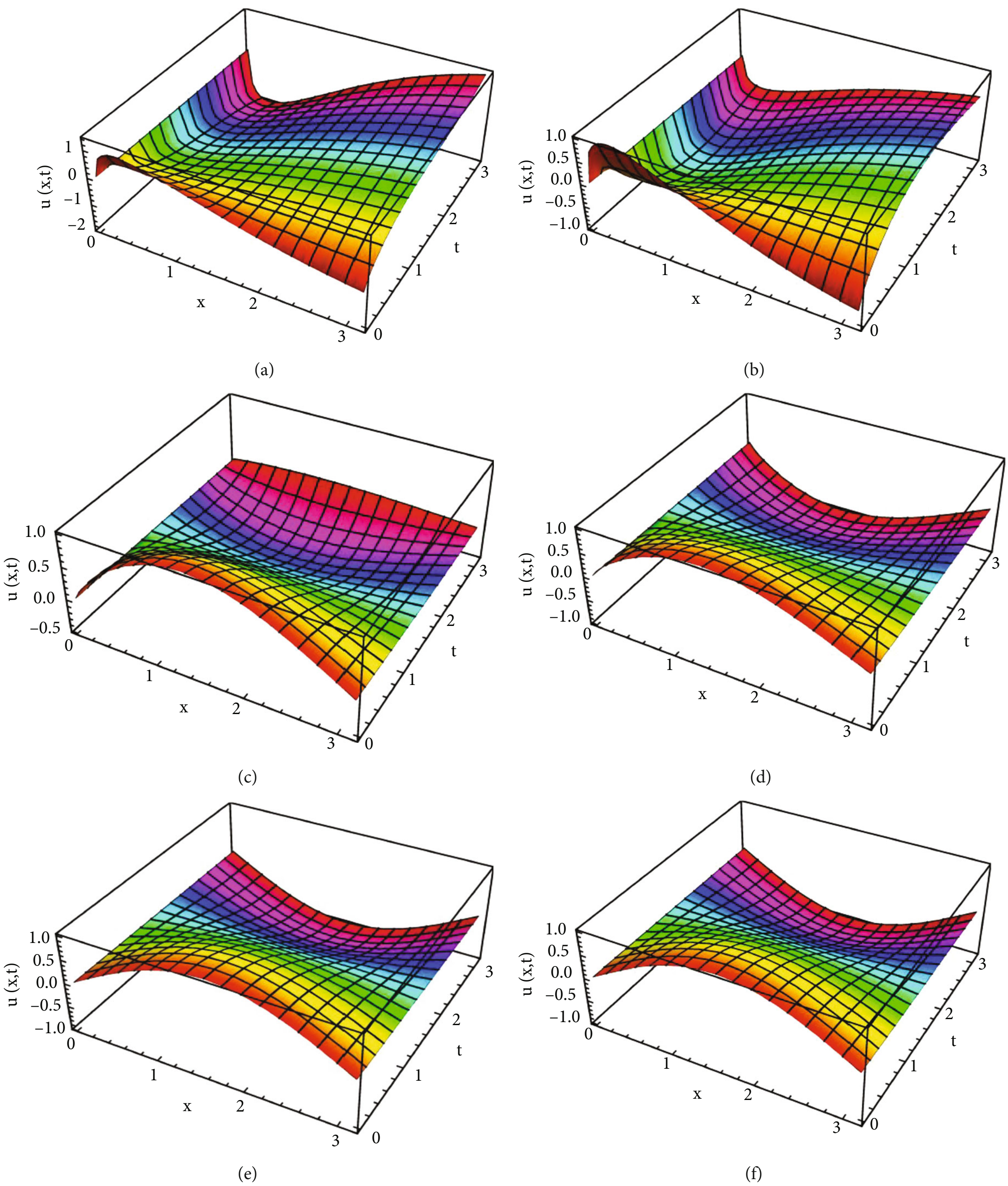


Figure 4: 3D solution plots of Example 4 obtained by the present method (a, c, e) in comparison with the exact solutions (b, d, f) at $\alpha = \beta = 0.4$, 0.8, and 1.

Liouville (ABR) sense of a function $f(\xi)$ are given as follows:

$$\begin{cases} {}_{a}^{ABC}D_{\xi}^{\mu}[f(\xi)] = \dfrac{B(\mu)}{1-\mu}\displaystyle\int_{a}^{\xi} f'(\lambda)E_{\mu}\left[\dfrac{-\mu}{1-\mu}(\xi-\lambda)^{\mu}\right]d\lambda, \\ {}_{a}^{ABR}D_{\xi}^{\mu}[f(\xi)] = \dfrac{B(\mu)}{1-\mu}\dfrac{d}{d\xi}\displaystyle\int_{a}^{\xi} f(\lambda)E_{\mu}\left[\dfrac{-\mu}{1-\mu}(\xi-\lambda)^{\mu}\right]d\lambda. \end{cases} \tag{7}$$

Definition 5 (see [51, 53, 54]). Given a function $f : \mathbb{R} \times (0, \infty) \longrightarrow \mathbb{R}$. Then, the conformable partial fractional derivatives (CPFDs) of $f(x, t)$ having order α and β are defined by

$$\begin{cases} \partial_{x}^{\alpha} f = \dfrac{\partial^{\alpha}}{\partial x^{\alpha}} f(x,t) = \lim\limits_{h\longrightarrow 0} \dfrac{f\left(x + hx^{1-\alpha}, t\right) - f(x,t)}{h}, \\ \partial_{t}^{\beta} f = \dfrac{\partial^{\beta}}{\partial t^{\beta}} f(x,t) = \lim\limits_{\varepsilon\longrightarrow 0} \dfrac{f\left(x, t + \varepsilon t^{1-\beta}\right) - f(x,t)}{\varepsilon}, \end{cases} \tag{8}$$

where $0 < \alpha, \beta \leq 1, x, t > 0$, and $\partial_x^\alpha f = (\partial^\alpha / \partial x^\alpha) f(x,t)$ and $\partial_t^\beta f = (\partial^\beta / \partial t^\beta) f(x,t)$ are called the fractional partial derivatives of orders α and β, respectively.

Theorem 6 (see [21, 23, 55]). *Let $\alpha, \beta \in (0, 1]$ and $f(x,t)$ be a differentiable at a point for $x, t > 0$. Then,\scale90%{*

$$\begin{cases} (i) \partial_x^\alpha f(x,t) = \dfrac{\partial^\alpha}{\partial x^\alpha} f(x,t) = x^{1-\alpha} \dfrac{\partial}{\partial x} f(x,t), \\ (ii)\ \partial_t^\beta f(x,t) = \dfrac{\partial^\beta}{\partial t^\beta} f(x,t) = t^{1-\beta} \dfrac{\partial}{\partial t} f(x,t). \end{cases} \tag{9}$$

Proof. (i) By definition of CFPD, we have

$$\partial_x^\alpha f = \frac{\partial^\alpha}{\partial x^\alpha} f(x,t) = \lim_{h \longrightarrow 0} \frac{f\left(x + h x^{1-\alpha}, t\right) - f(x,t)}{h}. \tag{10}$$

Using $\lambda = h x^{1-\alpha}$, we get

$$\begin{aligned} \partial_x^\alpha f = \frac{\partial^\alpha}{\partial x^\alpha} f(x,t) &= \lim_{\lambda \longrightarrow 0} \frac{f(x+\lambda, t) - f(x,t)}{\lambda x^{\alpha-1}} \\ &= x^{1-\alpha} \lim_{\lambda \longrightarrow 0} \frac{f(x+\lambda, t) - f(x,t)}{\lambda} = x^{1-\alpha} \frac{\partial}{\partial x} f(x,t). \end{aligned} \tag{11}$$

Similarly, we can prove the result of (ii). □

Lemma 7 (see [21, 40, 56]). *Let $\alpha, \beta \in (0, 1]$ and $(x,t), g(x,t) : \mathbb{R} \times [0, \infty) \longrightarrow \mathbb{R}$, be partially α, β-differentiable at $x, t > 0$ and c, d be real numbers. Then,*

$$\begin{aligned}
&\text{(i) } \frac{\partial^\alpha}{\partial x^\alpha}(c f(x,t) + d g(x,t)) = c \frac{\partial^\alpha}{\partial x^\alpha} f(x,t) + d \frac{\partial^\alpha}{\partial x^\alpha} g(x,t), \\
&\text{(ii) } \frac{\partial^\alpha}{\partial x^\alpha}(f g)(x,t) = g(x,t) \frac{\partial^\alpha}{\partial x^\alpha} f(x,t) + f(x,t) \frac{\partial^\alpha}{\partial x^\alpha} g(x,t), \\
&\text{(iii) } \frac{\partial^\alpha}{\partial x^\alpha}\left(\frac{f}{g}\right)(x,t) = \frac{g(x,t)(\partial^\alpha/\partial x^\alpha) f(x,t) + f(x,t)(\partial^\alpha/\partial x^\alpha) g(x,t)}{(g(x,t))^2}, \quad \text{provided that } g(x,t) \neq 0, \\
&\text{(iv) } \frac{\partial^\beta}{\partial t^\beta} f(x,t) = 0, \quad \text{if } f(x,t) \text{ was a function depending only on } x, \\
&\text{(v) } \frac{\partial^{\alpha+\beta}}{\partial x^\alpha \partial t^\beta}(x^m t^n) = m n x^{m-\alpha} t^{n-\beta}, \\
&\text{(vi) } \frac{\partial^\alpha}{\partial x^\alpha}\left(\left(\frac{x^\alpha}{\alpha}\right)^m \left(\frac{t^\beta}{\beta}\right)^n\right) = m \left(\frac{x^\alpha}{\alpha}\right)^{m-1} \left(\frac{t^\beta}{\beta}\right)^n, \frac{\partial^\beta}{\partial t^\beta}\left(\left(\frac{x^\alpha}{\alpha}\right)^m \left(\frac{t^\beta}{\beta}\right)^n\right) = n \left(\frac{x^\alpha}{\alpha}\right)^m \left(\frac{t^\beta}{\beta}\right)^{n-1}, \\
&\text{(vii) } \frac{\partial^\alpha}{\partial x^\alpha}\left(e^{c(x^\alpha/\alpha) + d\left(t^\beta/\beta\right)}\right) = c e^{c(x^\alpha/\alpha) + d\left(t^\beta/\beta\right)}, \frac{\partial^\beta}{\partial t^\beta}\left(e^{c(x^\alpha/\alpha) + d\left(t^\beta/\beta\right)}\right) = d e^{c(x^\alpha/\alpha) + d\left(t^\beta/\beta\right)}, \\
&\text{(viii) } \frac{\partial^\alpha}{\partial x^\alpha} \sin\left(\frac{x^\alpha}{\alpha}\right) \cos\left(\frac{t^\beta}{\beta}\right) = \cos\left(\frac{x^\alpha}{\alpha}\right) \cos\left(\frac{t^\beta}{\beta}\right), \frac{\partial^\beta}{\partial t^\beta} \sin\left(\frac{x^\alpha}{\alpha}\right) \cos\left(\frac{t^\beta}{\beta}\right) = -\sin\left(\frac{x^\alpha}{\alpha}\right) \sin\left(\frac{t^\beta}{\beta}\right).
\end{aligned} \tag{12}$$

Definition 8 (see [43, 57]). Let $0 < \alpha, \beta \leq 1, x, t > 0$ and $f : [a, \infty) \times [b, \infty) \longrightarrow \mathbb{R}$. Then, the conformable partial fractional derivatives (CFPDs) of $f(x,t)$ having orders α and β are defined by

$$\begin{cases} \dfrac{\partial^\alpha}{\partial x^\alpha} f(x,t) = \lim\limits_{h \longrightarrow 0} \dfrac{f\left(x + h(x-a)^{1-\alpha}, t\right) - f(x,t)}{h}, \\ \dfrac{\partial^\beta}{\partial t^\beta} f(x,t) = \lim\limits_{\varepsilon \longrightarrow 0} \dfrac{f\left(x, t + \varepsilon(t-b)^{1-\beta}\right) - f(x,t)}{\varepsilon}. \end{cases} \tag{13}$$

Remark 9. All properties in Lemma 7 are valid also for Definition 8 when $(x-a)$ is placed instead of x and $(t-b)$ is placed instead of t.

3. Reduced Differential Transform Method (RDTM)

In this subsection, some basic RDTM terminology and theorems are explained briefly for the conformable fractional differential equations (CFDE). For more information, read [20, 21, 28, 59] and the references therein.

Definition 10. Assume that $u(x,t)$ is infinitely partially α-differentiable around zero with respect to t. Then, the RDT of $u(x,t)$ is given as

$$U_k^\alpha(x)=\frac{1}{\alpha^k k!}\left[\left(\partial_t^\alpha\right)^k u(x,t)\right]_{t=t_0}, \tag{14}$$

where $(\partial_t^\alpha)^k u(x,t)=\underbrace{(\partial_t^\alpha\partial_t^\alpha\cdots\partial_t^\alpha)}_{k\text{-times}}u(x,t)$, and k is a nonnegative integer.

Definition 11. Given that $U_k^\alpha(x)$ is the RDT of $u(x,t)$. Then, the inverse RDT of $U_k^\alpha(x)$ is given by

$$u(x,t)=\sum_{k=0}^{\infty}U_k^\alpha(x)(t-t_0)^{\alpha k}=\sum_{k=0}^{\infty}\frac{1}{\alpha^k k!}\left(\partial_t^\alpha\right)^k u(x,t)(t-t_0)^{\alpha k}, \tag{15}$$

where the RDT of $u(x,t)$ at the initial conditions is defined as

$$U_k^\alpha(x)=\begin{cases}\dfrac{1}{(\alpha k)!}\left[\dfrac{\partial^{\alpha k}}{\partial t^{\alpha k}}u(x,t)\right]_{t=t_0}, & \text{if } k\alpha\in\mathbb{Z}^+,\\ 0, & \text{if } k\alpha\notin\mathbb{Z}^+,\end{cases}\quad \text{for } k=0,1,\cdots,\left(\frac{n}{\alpha}-1\right), \tag{16}$$

where "n" is the order of the conformable fractional PDE.

By consideration of $U_0^\alpha(x)=f(x)$ as a transformation of the initial condition

$$u(x,0)=f(x). \tag{17}$$

A straightforward iterative calculation gives the $U_k^\alpha(x)$ values for $k=1,2,3,\cdots,n$.

Then, the inverse transformation of the $\{U_k^\alpha(x)\}_{k=0}^n$ gives the approximate solution as

$$\tilde{u}_n(x,t)=\sum_{k=0}^{n}U_k^\alpha(x)t^{\alpha k}, \tag{18}$$

where "n" represents the order of the obtained approximation solution.

Therefore, the RDTM leads a solution as follows:

$$u(x,t)=\lim_{n\longrightarrow\infty}\tilde{u}_n(x,t). \tag{19}$$

Theorem 12 (see [58]). *If $f(t)=e^{\lambda((t-t_0)^\alpha/\alpha)}$, then $F_\alpha(k)=\lambda^k/\alpha^k k!$, where λ is constant.*

Proof. RDT of a function f is

$$F_\alpha(k)=\frac{1}{\alpha^k k!}\left[\left(\partial_{t_0}^\alpha\left(e^{\lambda((t-t_0)^\alpha/\alpha)}\right)\right)^{(k)}(t)\right]_{t=t_0}. \tag{20}$$

Calculating the conformable fractional derivative of $f(t)=e^{\lambda((t-t_0)^\alpha/\alpha)}$ for k times, where $\alpha\in(0,1]$, we obtain $F_\alpha(k)=\lambda^k/\alpha^k k!$. □

Theorem 13 (see [58]). *If $u(x,t)=\sin(\gamma((x-x_0)^\beta/\beta)+\omega((t-t_0)^\alpha/\alpha))$, then $U_k^\alpha(x)=(\omega^k/\alpha^k k!)\sin(k(\pi/2)+\gamma((x-x_0)^\beta/\beta))$, and if $u(x,t)=\cos(\gamma((x-x_0)^\beta/\beta)+\omega((t-t_0)^\alpha/\alpha))$, for $\alpha\in(0,1)$, then $U_k^\alpha(x)=(\omega^k/\alpha^k k!)\cos(k(\pi/2)+\gamma((x-x_0)^\beta/\beta))$, where ω and γ are constants.*

Proof. RDT of $u(x,t)=\sin(\gamma((x-x_0)^\beta/\beta)+\omega((t-t_0)^\alpha/\alpha))$ is written as

$$U_k^\alpha(x)=\frac{1}{\alpha^k k!}\left[\left(\partial_{t_0}^\alpha\left(\sin\left(\gamma\frac{(x-x_0)^\beta}{\beta}+\omega\frac{(t-t_0)^\alpha}{\alpha}\right)\right)\right)^{(k)}(t)\right]_{t=t_0}. \tag{21}$$

Thus, it is obtained that

$$U_k^\alpha(x)=\frac{\omega^k}{\alpha^k k!}\sin\left(k\frac{\pi}{2}+\gamma\frac{(x-x_0)^\beta}{\beta}\right). \tag{22}$$

Similarly, the other part is proven. □

Theorem 14. *If $w(x,t)=u^2(x,t)$, then $W_k^\alpha(x)=\sum_{r=0}^k U_r^\alpha(x)U_{k-r}^\alpha(x)$.*

Proof. With the help of Definition 11, $u(x,t)$ can be written as

$$u(x,t)=\sum_{k=0}^{\infty}U_k^\alpha(x)(t-t_0)^{k\alpha}. \tag{23}$$

Then, $w(x,t)$ is obtained as

$$\begin{aligned}w(x,t)=u^2(x,t)&=\sum_{k=0}^{\infty}U_k^\alpha(x)(t-t_0)^{k\alpha}\sum_{k=0}^{\infty}U_k^\alpha(x)(t-t_0)^{k\alpha}\\&=\left(U_0^\alpha(x)+U_1^\alpha(x)(t-t_0)^\alpha+U_2^\alpha(x)(t-t_0)^{2\alpha}\right.\\&\quad\left.+U_3^\alpha(x)(t-t_0)^{3\alpha}+U_4^\alpha(x)(t-t_0)^{4\alpha}+\cdots\right)\\&\quad\times\left(U_0^\alpha(x)+U_1^\alpha(x)(t-t_0)^\alpha+U_2^\alpha(x)(t-t_0)^{2\alpha}\right.\\&\quad\left.+U_3^\alpha(x)(t-t_0)^{3\alpha}+U_4^\alpha(x)(t-t_0)^{4\alpha}+\cdots\right)\\&=U_0^\alpha(x)U_0^\alpha(x)+\left(U_0^\alpha(x)U_1^\alpha(x)\right.\\&\quad\left.+U_1^\alpha(x)U_0^\alpha(x)\right)(t-t_0)^\alpha+\left(U_0^\alpha(x)U_2^\alpha(x)\right.\\&\quad\left.+U_1^\alpha(x)U_1^\alpha(x)+U_2^\alpha(x)U_0^\alpha(x)\right)(t-t_0)^{2\alpha}+\cdots\\&=\sum_{k=0}^{\infty}\sum_{r=0}^{k}U_r^\alpha(x)U_{k-r}^\alpha(x)(t-t_0)^{k\alpha}.\end{aligned} \tag{24}$$

Table 5: The fundamental mathematical operations of RDT [37, 39, 40, 42].

Original function	Transformed function
$u(x,t)$	$U_k^\alpha(x) = 1/\alpha^k k! \left[(\partial_t^\alpha)^k u(x,t)\right]_{t=t_0}$
$u(x,t) = av(x,t) \pm bw(x,t)$	$U_k^\alpha(x) = aV_k^\alpha(x) \pm bW_k^\alpha(x)$
$u(x,t) = v(x,t)w(x,t)$	$U_k^\alpha(x) = \sum_{r=0}^{k} V_r^\alpha(x) W_{k-r}^\alpha(x)$
$u(x,t) = (\partial^\alpha/\partial t^\alpha)v(x,t)$	$U_k^\alpha(x) = \alpha(k+1)V_{k+1}^\alpha(x)$
$u(x,t) = x^m(t-t_0)^n$	$U_k^\alpha(x) = x^m \delta(k-n/\alpha)$, where $\delta(k-n/\alpha) = \begin{cases} 1, & \text{if } k = n/\alpha, \\ 0, & \text{if } k \neq n/\alpha. \end{cases}$

Hence, $W_k^\alpha(x)$ is found as

$$W_k^\alpha(x) = \sum_{r=0}^{k} U_r^\alpha(x) U_{k-r}^\alpha(x). \tag{25}$$

The proof is completed. □

Lemma 15. *If $w(x,t) = u^3(x,t)$, then $W_k^\alpha(x) = \sum_{r=0}^{k}\sum_{i=0}^{r} U_i^\alpha(x) U_{r-i}^\alpha(x) U_{k-r}^\alpha(x)$.*

Proof. Observe that $u^3(x,t) = u^2(x,t)u(x,t)$.

So, by using Definition 8 and Table 5, we have

$$\begin{aligned} W_k^\alpha(x) &= \frac{1}{\alpha^k k!}\left[(\partial_t^\alpha)^k u^3(x,t)\right]_{t=t_0} \\ &= \frac{1}{\alpha^k k!}\left[(\partial_t^\alpha)^k u^2(x,t)u(x,t)\right]_{t=t_0} = \sum_{r=0}^{k} F_r^\alpha(x) U_{k-r}^\alpha(x), \end{aligned} \tag{26}$$

where $F_r^\alpha(x)$ is the reduced differential transform of $u^2(x,t)$; i.e., by using Theorem 14,

$$F_r^\alpha(x) = \sum_{i=0}^{r} U_i^\alpha(x) U_{r-i}^\alpha(x). \tag{27}$$

Thus,

$$W_k^\alpha(x) = \sum_{r=0}^{k} F_r^\alpha(x) U_{k-r}^\alpha(x) = \sum_{r=0}^{k}\sum_{i=0}^{r} U_i^\alpha(x) U_{r-i}^\alpha(x) U_{k-r}^\alpha(x). \tag{28}$$

The proof is completed. □

Theorem 16. *If $w(x,t) = \partial^{2\alpha}u(x,t)/\partial t^{2\alpha}$, then $W_k^\alpha(x) = \alpha^2(k+2)(k+1)U_{k+2}^\alpha(x)$.*

Proof. With the help of Definition 10, RDT of $u(x,t)$ can be written as

$$U_k^\alpha(x) = \frac{1}{\alpha^k k!}\left[(\partial_t^\alpha)^k u(x,t)\right]_{t=t_0}. \tag{29}$$

For $w(x,t) = \partial^{2\alpha}u(x,t)/\partial t^{2\alpha}$,

$$\begin{aligned} W_k^\alpha(x) &= \frac{1}{\alpha^k k!}\left[(\partial_t^\alpha)^k \frac{\partial^{2\alpha}u(x,t)}{\partial t^{2\alpha}}\right]_{t=t_0} = \frac{1}{\alpha^k k!}\left[(\partial_t^\alpha)^{k+2} u(x,t)\right]_{t=t_0} \\ &= \alpha^2(k+2)(k+1)\underbrace{\frac{1}{\alpha^{k+2}(k+2)!}\left[(\partial_t^\alpha)^{k+2}u(x,t)\right]_{t=t_0}}_{U_{k+2}^\alpha(x)} \\ &= \alpha^2(k+2)(k+1)U_{k+2}^\alpha(x). \end{aligned} \tag{30}$$

Hence, the theorem is proved. □

4. RDTM to Solve Conformable Fractional Telegram Equations

To illustrate the basic concepts of the RDTM, we consider the one-dimensional conformable fractional telegraph equation (2) with initial conditions (3) and (4).

We develop the following iteration formula based on the RDTM definition and properties in Table 5:

$$\begin{aligned} \alpha^2(k+2)(k+1)U_{k+2}^\alpha(x) = {} & c\frac{\partial^\beta}{x^\beta}U_k^\alpha(x) - a\alpha(k+1)U_{k+1}^\alpha(x) \\ & - bN_k^\alpha(x) + F_k^\alpha(x), \end{aligned} \tag{31}$$

where $U_k^\alpha(x), N_k^\alpha(x)$, and $F_k^\alpha(x)$ are the reduced differential transform functions of $u(x,t), N(u(x,t))$, and $f(x,t)$, respectively, in view of the conformable fractional derivative.

Solving for $U_{k+2}^\alpha(x)$ in equation above, we obtain

$$\begin{aligned} U_{k+2}^\alpha(x) = {} & \frac{1}{\alpha^2(k+2)(k+1)} \\ & \cdot \left[c\frac{\partial^\beta}{x^\beta}U_k^\alpha(x) - a\alpha(k+1)U_{k+1}^\alpha(x) - bN_k^\alpha(x) + F_k^\alpha(x)\right]. \end{aligned} \tag{32}$$

Applying the properties of RDTM to initial conditions (2) and (3) in the sense of equation (16), we get

$$U_0^\alpha(x) = g_0(x), \tag{33}$$

$$U_1^\alpha(x) = \frac{1}{\alpha} g_1(x). \tag{34}$$

Plugging (34) and (33) into equation (32), and by straightforward iterative calculations, we get the remaining successive values of $U_k^\alpha(x)$, as follows:

$$\begin{aligned} U_2^\alpha(x) &= \frac{1}{2!\alpha^2}\left(c\frac{\partial^\beta}{x^\beta} g_0(x) - a g_1(x) - bN_0^\alpha(x) + F_0^\alpha(x)\right), \\ U_3^\alpha(x) &= \frac{1}{3!\alpha^2}\left(\frac{c}{\alpha}\frac{\partial^\beta}{x^\beta} g_1(x) - 2a\alpha U_2^\alpha(x) - bN_1^\alpha(x) + F_1^\alpha(x)\right), \end{aligned} \tag{35}$$

and so on.

Then, the inverse reduced differential transform of the set of values $\{U_k^\alpha(x)\}_{k=0}^n$ gives the n-term approximate solution as

$$\tilde{u}_n(x,t) = \sum_{k=0}^{n} U_k^\alpha(x) t^{k\alpha}. \tag{36}$$

Therefore, the exact solution to the problem (2) is given by

$$\begin{aligned} u(x,t) &= \lim_{n\longrightarrow\infty} \tilde{u}_n(x,t) = \sum_{k=0}^{\infty} U_k^\alpha(x) t^{k\alpha} = U_0^\alpha(x) + U_1^\alpha(x) t^\alpha \\ &+ U_2^\alpha(x) t^{2\alpha} + \cdots = g_0(x) + \left[\frac{1}{\alpha} g_1(x)\right] t^\alpha \\ &+ \left[\frac{1}{2!\alpha^2}\left(c\frac{\partial^\beta}{x^\beta} g_0(x) - a g_1(x) - bN_0^\alpha(x) + F_0^\alpha(x)\right)\right] t^{2\alpha} \\ &+ \left[\frac{1}{3!\alpha^3}\left(\frac{c}{\alpha}\frac{\partial^\beta}{x^\beta} g_1(x) - 2a\alpha U_2^\alpha(x) - bN_1^\alpha(x) + F_1^\alpha(x)\right)\right] t^{3\alpha} + . \cdots \end{aligned} \tag{37}$$

5. Illustrative Examples

To check the accuracy and efficiency of the RDTM, we consider four testing examples in this section.

5.1. Analytical Solution to Illustrative Examples. In this subsection, the proposed method RDTM was applied to obtain the analytical solution to each illustrative example.

Example 1 (see [49, 50]). Consider the conformable fractional space-time telegraph equation stated as

$$\frac{\partial^{2\alpha} u(x,t)}{\partial t^{2\alpha}} + \frac{\partial^\alpha u(x,t)}{\partial t^\alpha} = \frac{\partial^{2\beta} u(x,t)}{\partial x^{2\beta}} - u(x,t) + 2e^{x^\beta/\beta + t^\alpha/\alpha}, \tag{38}$$

with initial conditions

$$u(x,0) = e^{x^\beta/\beta}, \tag{39}$$

$$u_t(x,0) = e^{x^\beta/\beta}. \tag{40}$$

We develop the following recursive formula by applying the properties of reduced differential transform to both sides of equation (38):

$$\begin{aligned} U_{k+2}^\alpha(x) &= \frac{1}{\alpha^2(k+2)(k+1)} \\ &\cdot \left[-\alpha(k+1)U_{k+1}^\alpha(x) + \frac{\partial^{2\beta}}{\partial x^{2\beta}} U_k^\alpha(x) - U_k^\alpha(x) + 2e^{x^\beta/\beta}\frac{(1)^k}{\alpha^k k!}\right]. \end{aligned} \tag{41}$$

Taking the properties of reduced differential transform to initial conditions (39) and (40), we get

$$U_0^\alpha(x) = e^{x^\beta/\beta}, \tag{42}$$

$$U_1^\alpha(x) = \frac{e^{x^\beta/\beta}}{\alpha}. \tag{43}$$

Substituting equations (43) and (42) into equation (41), for $k = 0, 1, 2, \cdots$, we obtain the following successive iterated values:

$$\begin{aligned} U_2^\alpha(x) &= \frac{e^{x^\beta/\beta}}{2!\alpha^2}, U_3^\alpha(x) = \frac{e^{x^\beta/\beta}}{3!\alpha^3}, U_4^\alpha(x) = \frac{e^{x^\beta/\beta}}{4!\alpha^4}, U_5^\alpha(x) \\ &= \frac{e^{x^\beta/\beta}}{5!\alpha^5}, \cdots, U_k^\alpha(x) = \frac{e^{x^\beta/\beta}}{k!\alpha^k}. \end{aligned} \tag{44}$$

We acquire the following results by using the inverse reduced differential transform (15):

$$\begin{aligned} u(x,t) &= \sum_{k=0}^{\infty} U_k^\alpha(x) t^{k\alpha} = U_0^\alpha(x) + U_1^\alpha(x) t^\alpha + U_2^\alpha(x) t^{2\alpha} \\ &+ U_3^\alpha(x) t^{3\alpha} + U_4^\alpha(x) t^{4\alpha} + \cdots = e^{x^\beta/\beta} + \frac{e^{x^\beta/\beta}}{\alpha} t^\alpha \\ &+ \frac{e^{x^\beta/\beta}}{2!\alpha^2} t^{2\alpha} + \frac{e^{x^\beta/\beta}}{3!\alpha^3} t^{3\alpha} + \frac{e^{x^\beta/\beta}}{4!\alpha^4} t^{4\alpha} + \frac{e^{x^\beta/\beta}}{5!\alpha^5} t^{5\alpha} \\ &+ \cdots = e^{x^\beta/\beta}\left(1 + \frac{t^\alpha}{\alpha} + \frac{t^{2\alpha}}{2!\alpha^2} + \frac{t^{3\alpha}}{3!\alpha^3} + \frac{t^{4\alpha}}{4!\alpha^4} + \frac{t^{5\alpha}}{5!\alpha^5} + \cdots\right) \\ &= e^{x^\beta/\beta} \sum_{k=0}^{\infty} \frac{t^{k\alpha}}{k!\alpha^k} = e^{x^\beta/\beta + t^\alpha/\alpha}. \end{aligned} \tag{45}$$

Remark 17. For $\alpha = \beta = 1$, equation (38) reduces to the classical (or nonfractional) space-time telegraph equation, and its exact solution is given by $u(x,t) = e^{x+t}$..

Example 2 (see [59, 60]). Consider the following time fractional-order nonlinear telegraph equation with the external source term

$$\frac{\partial^{2\alpha} u}{\partial t^{2\alpha}} + 2\frac{\partial^\alpha u}{\partial t^\alpha} + u^2 = \frac{\partial^{2\beta} u}{\partial x^{2\beta}} + e^{2\left(x^\beta/\beta\right) - 4\left(t^\alpha/\alpha\right)} - e^{x^\beta/\beta - 2\left(t^\alpha/\alpha\right)}, \tag{46}$$

with the initial conditions

$$u(x,0) = e^{x^{\beta}/\beta}, \tag{47}$$

$$u_t(x,0) = -2e^{x^{\beta}/\beta}. \tag{48}$$

On both sides of equation (46), using the properties of the reduced differential transform method, we get

$$\begin{aligned} U_{k+2}^{\alpha}(x) = -\frac{1}{\alpha^2(k+2)(k+1)}\Bigg[& 2\alpha(k+1)U_{k+1}^{\alpha}(x) \\ & + \sum_{r=0}^{k} U_r^{\alpha}(x)U_{k-r}^{\alpha}(x) - \frac{\partial^{2\beta}}{\partial x^{2\beta}}U_k^{\alpha}(x) \\ & - e^{2\left(x^{\beta}/\beta\right)}\frac{(-4)^k}{\alpha^k k!} + e^{x^{\beta}/\beta}\frac{(-2)^k}{\alpha^k k!}\Bigg]. \end{aligned} \tag{49}$$

Applying the properties of reduced differential transform to initial conditions (47) and (48), we get

$$U_0^{\alpha}(x) = e^{x^{\beta}/\beta}, \tag{50}$$

$$U_1^{\alpha}(x) = -2\frac{e^{x^{\beta}/\beta}}{\alpha}. \tag{51}$$

Plugging equations (50) and (51) into equation (49), for $k = 0, 1, 2, \cdots$, we obtain the following successive iterated values:

$$\begin{aligned} U_2^{\alpha}(x) &= (-2)^2\frac{e^{x^{\beta}/\beta}}{2!\alpha^2}, U_3^{\alpha}(x) = (-2)^3\frac{e^{x^{\beta}/\beta}}{3!\alpha^3}, U_4^{\alpha}(x) \\ &= (-2)^4\frac{e^{x^{\beta}/\beta}}{4!\alpha^4}, \cdots, U_k^{\alpha}(x) = (-2)^k\frac{e^{x^{\beta}/\beta}}{k!\alpha^k}. \end{aligned} \tag{52}$$

Applying the inverse reduced differential transform, we get

$$\begin{aligned} u(x,t) = \sum_{k=0}^{\infty} U_k^{\alpha}(x)t^{k\alpha} &= U_0^{\alpha}(x) + U_1^{\alpha}(x)t^{\alpha} + U_2^{\alpha}(x)t^{2\alpha} \\ &+ U_3^{\alpha}(x)t^{3\alpha} + U_4^{\alpha}(x)t^{4\alpha} + \cdots = e^{x^{\beta}/\beta} + (-2)\frac{e^{x^{\beta}/\beta}}{\alpha}t^{\alpha} \\ &+ (-2)^2\frac{e^{x^{\beta}/\beta}}{2!\alpha^2}t^{2\alpha} + (-2)^3\frac{e^{x^{\beta}/\beta}}{3!\alpha^3}t^{3\alpha} + (-2)^4\frac{e^{x^{\beta}/\beta}}{4!\alpha^4}t^{4\alpha} \\ &+ (-2)^2\frac{e^{x^{\beta}/\beta}}{5!\alpha^5}t^{5\alpha} + \cdots = e^{x^{\beta}/\beta}\left(1 + (-2)\frac{t^{\alpha}}{\alpha} + (-2)^2\frac{t^{2\alpha}}{2!\alpha^2}\right. \\ &\left. + (-2)^3\frac{t^{3\alpha}}{3!\alpha^3} + (-2)^4\frac{t^{4\alpha}}{4!\alpha^4} + (-2)^5\frac{t^{5\alpha}}{5!\alpha^5} + \cdots\right) \\ = e^{x^{\beta}/\beta}\sum_{k=0}^{\infty}\frac{(-2t^{\alpha})^k}{k!\alpha^k} &= e^{x^{\beta}/\beta - 2(t^{\alpha}/\alpha)}. \end{aligned} \tag{53}$$

Remark 18. For $\alpha = \beta = 1$, equation (46) reduces to the classical (or nonfractional) space-time telegraph equation, and its exact solution is given by $u(x,t) = e^{x-2t}$.

Example 3 (see [60]). Consider the conformable fractional nonlinear telegraph equation with the external source term as follows:

$$\frac{\partial^{2\alpha}u}{\partial t^{2\alpha}} + 4\frac{\partial^{\alpha}u}{\partial t^{\alpha}} + u^3 = \frac{\partial^{2\beta}u}{\partial x^{2\beta}} - 2e^{-t^{\alpha}/\alpha}\sin\left(\frac{x^{\beta}}{\beta}\right) + e^{-3(t^{\alpha}/\alpha)}\sin^3\left(\frac{x^{\beta}}{\beta}\right), \tag{54}$$

with the initial conditions

$$u(x,0) = \sin\left(\frac{x^{\beta}}{\beta}\right), \tag{55}$$

$$u_t(x,0) = -\sin\left(\frac{x^{\beta}}{\beta}\right). \tag{56}$$

Applying the properties of the reduced differential transform method on both sides of equation (54), we construct the following recursive formula:

$$\begin{aligned} U_{k+2}^{\alpha}(x) = -\frac{1}{\alpha^2(k+2)(k+1)} \cdot \left[\begin{matrix} 4\alpha(k+1)U_{k+1}^{\alpha}(x) + \displaystyle\sum_{r=0}^{k}\sum_{i=0}^{r}U_i^{\alpha}(x)U_{r-i}^{\alpha}(x)U_{k-r}^{\alpha}(x) \\ -\dfrac{\partial^{2\beta}}{\partial x^{2\beta}}U_k^{\alpha}(x) - \dfrac{(-1)^k}{\alpha^k k!}\sin\left(\dfrac{x^{\beta}}{\beta}\right) - \dfrac{(-3)^k}{\alpha^k k!}\sin\left(\dfrac{x^{\beta}}{\beta}\right) \end{matrix}\right]. \end{aligned} \tag{57}$$

Applying the properties of reduced differential transform to initial conditions (55) and (56), we get

$$U_0^{\alpha}(x) = \sin\left(\frac{x^{\beta}}{\beta}\right), \tag{58}$$

$$U_1^{\alpha}(x) = -\frac{\sin\left(x^{\beta}/\beta\right)}{\alpha}. \tag{59}$$

Substituting equations (59) and (58) into equation (57), for $k = 0, 1, 2, \cdots$, we obtain the following successive iterated values:

$$\begin{aligned} U_2^{\alpha}(x) &= \frac{\sin\left(x^{\beta}/\beta\right)}{2!\alpha^2}, U_3^{\alpha}(x) = -\frac{\sin\left(x^{\beta}/\beta\right)}{3!\alpha^3}, U_4^{\alpha}(x) \\ &= \frac{\sin\left(x^{\beta}/\beta\right)}{4!\alpha^4}, U_5^{\alpha}(x) = -\frac{\sin\left(x^{\beta}/\beta\right)}{5!\alpha^5}, \cdots, U_k^{\alpha}(x) \\ &= (-1)^k\frac{\sin\left(x^{\beta}/\beta\right)}{k!\alpha^k}. \end{aligned} \tag{60}$$

Applying the inverse reduced differential transform (15),

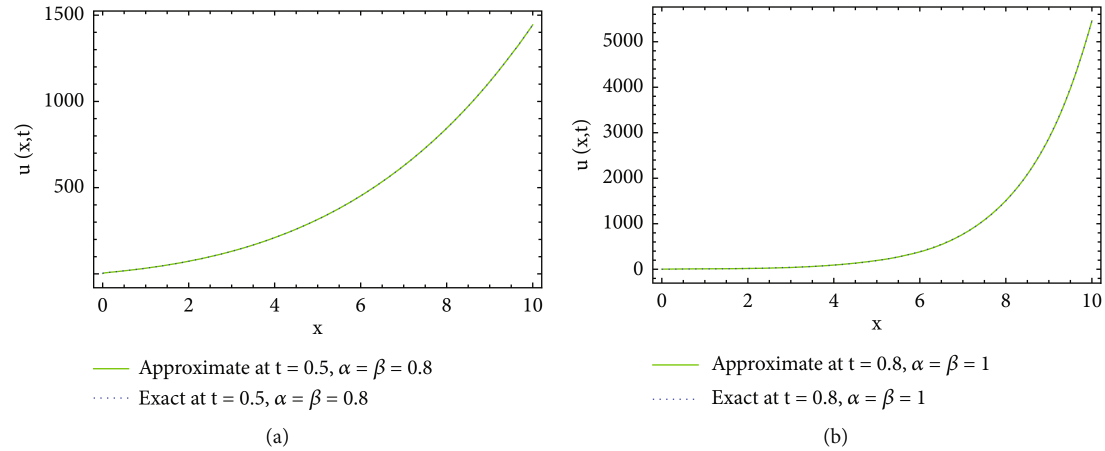


FIGURE 5: 2D solution plots of Example 1 obtained by the present method in comparison with the exact solutions for different values of fractional orders α and β.

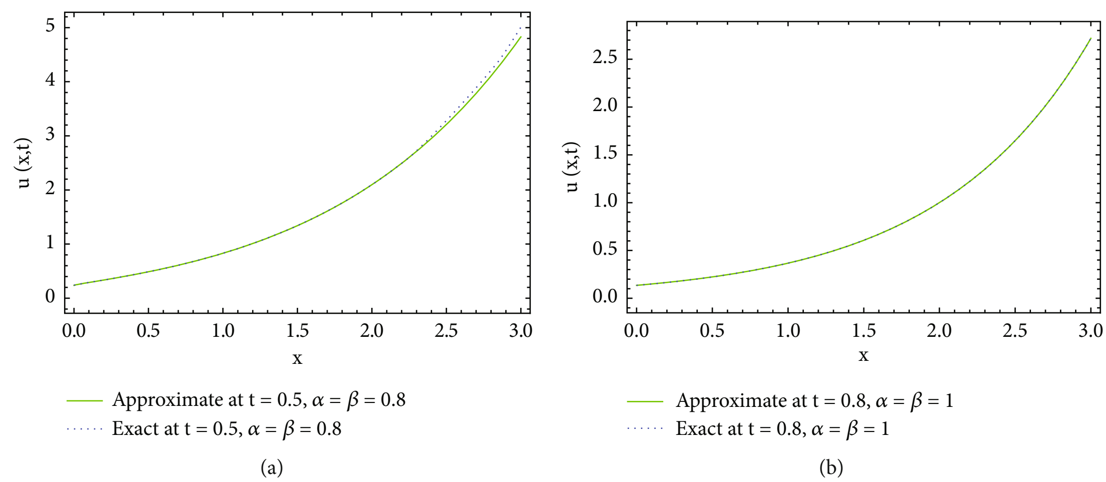


FIGURE 6: 2D solution plots of Example 2 obtained by the present method in comparison with the exact solutions for different values of fractional orders α and β.

we get

$$\begin{aligned}
u(x,t) &= \sum_{k=0}^{\infty} U_k^{\alpha}(x)t^{k\alpha} = U_0^{\alpha}(x) + U_1^{\alpha}(x)t^{\alpha} + U_2^{\alpha}(x)t^{2\alpha} \\
&\quad + U_3^{\alpha}(x)t^{3\alpha} + \cdots = \sin\left(\frac{x^{\beta}}{\beta}\right) - \frac{\sin\left(x^{\beta}/\beta\right)}{\alpha}t^{\alpha} \\
&\quad + \frac{\sin\left(x^{\beta}/\beta\right)}{2!\alpha^2}t^{2\alpha} - \frac{\sin\left(x^{\beta}/\beta\right)}{3!\alpha^3}t^{3\alpha} + \frac{\sin\left(x^{\beta}/\beta\right)}{4!\alpha^4}t^{4\alpha} \\
&\quad - \frac{\sin\left(x^{\beta}/\beta\right)}{5!\alpha^5}t^{5\alpha} + \cdots = \sin\left(\frac{x^{\beta}}{\beta}\right) \\
&\quad \cdot \left(1 - \frac{t^{\alpha}}{\alpha} + \frac{t^{2\alpha}}{2!\alpha^2} - \frac{t^{3\alpha}}{3!\alpha^3} + \frac{t^{4\alpha}}{4!\alpha^4} - \frac{t^{5\alpha}}{5!\alpha^5} + \cdots\right) \\
&= \sin\left(\frac{x^{\beta}}{\beta}\right)\sum_{k=0}^{\infty}\frac{(-t^{\alpha})^k}{k!\alpha^k} = e^{-t^{\alpha}/\alpha}\sin\left(\frac{x^{\beta}}{\beta}\right).
\end{aligned} \tag{61}$$

Remark 19. For $\alpha = \beta = 1$, equation (54) reduces to the classical (or nonfractional) space-time telegraph equation, and its exact solution is given by $u(x,t) = e^{-t}\sin x$..

Example 4 (see [47]). Consider the following conformable fractional nonlinear telegraph equation with the external source term given by

$$\begin{aligned}
\frac{\partial^{2\alpha}u}{\partial t^{2\alpha}} + 20\frac{\partial^{\alpha}u}{\partial t^{\alpha}} + 25u &= \frac{\partial^{2\beta}u}{\partial x^{2\beta}} - 20\sin\left(\frac{x^{\beta}}{\beta}\right)\sin\left(\frac{t^{\alpha}}{\alpha}\right) \\
&\quad + 25\sin\left(\frac{x^{\beta}}{\beta}\right)\cos\left(\frac{t^{\alpha}}{\alpha}\right),
\end{aligned} \tag{62}$$

with the initial conditions

$$u(x,0) = \sin\left(\frac{x^{\beta}}{\beta}\right), \tag{63}$$

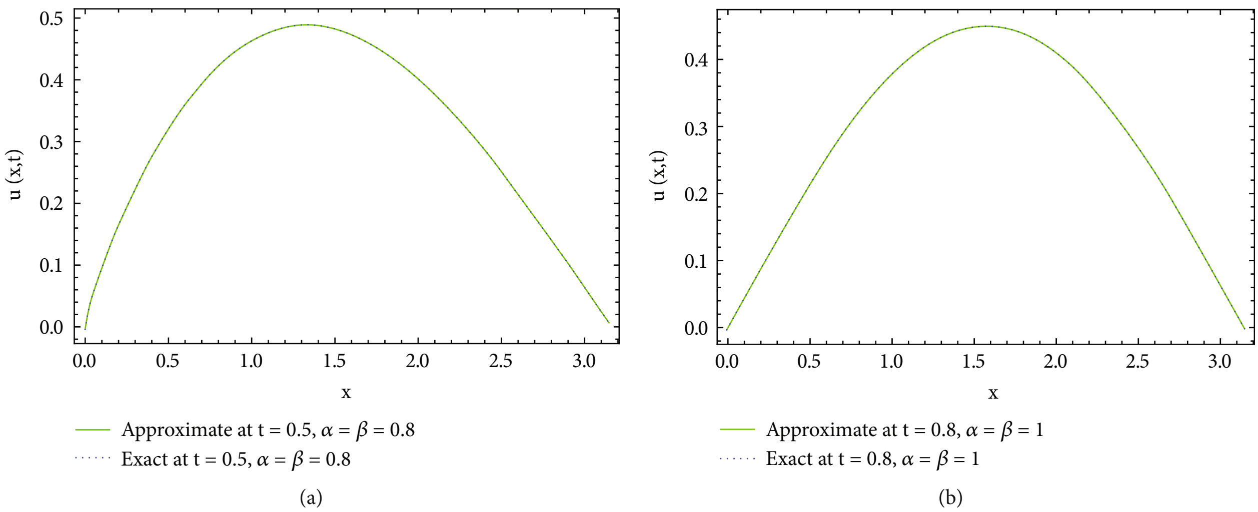


Figure 7: 2D solution plots of Example 3 obtained by the present method in comparison with the exact solutions for different values of fractional orders α and β.

$$u_t(x,0)=0. \tag{64}$$

Applying the properties of the reduced differential transform method on both sides of equation (62), we construct the following recursive formula:

$$\begin{aligned}&\alpha^2(k+2)(k+1)U_{k+2}^{\alpha}(x)+20\alpha(k+1)U_{k+1}^{\alpha}(x)+25U_k^{\alpha}(x)\\&=\frac{\partial^{2\beta}}{\partial x^{2\beta}}U_k^{\alpha}(x)-20\sin\left(\frac{x^{\beta}}{\beta}\right)\frac{(1)^k}{\alpha^k k!}\sin\left(k\frac{\pi}{2}\right)\\&\quad+25\sin\left(\frac{x^{\beta}}{\beta}\right)\frac{(1)^k}{\alpha^k k!}\cos\left(k\frac{\pi}{2}\right).\end{aligned} \tag{65}$$

Applying the properties of reduced differential transform to initial conditions (63) and (64), we obtain

$$U_0^{\alpha}(x)=\sin\left(\frac{x^{\beta}}{\beta}\right), \tag{66}$$

$$U_1^{\alpha}(x)=0. \tag{67}$$

Substituting equations (66) and (67) into equation (65), for $k=0,1,2,\cdots$, we obtain the following successive iterated values:

$$\begin{aligned}U_2^{\alpha}(x)&=-\frac{\sin\left(x^{\beta}/\beta\right)}{2!\alpha^2},\ U_3^{\alpha}(x)=0,\ U_4^{\alpha}(x)\\&=\frac{\sin\left(x^{\beta}/\beta\right)}{4!\alpha^4},\ U_5^{\alpha}(x)=0,\ U_6^{\alpha}(x)=-\frac{\sin\left(x^{\beta}/\beta\right)}{6!\alpha^6},\cdots,\end{aligned}$$

$$U_k^{\alpha}(x)=\begin{cases}(-1)^k\dfrac{\sin\left(x^{\beta}/\beta\right)}{(2k)!\alpha^k}, & \text{if } k \text{ is even},\\[2mm] 0, & \text{if } k \text{ is odd}.\end{cases} \tag{68}$$

We acquire the following results by using the inverse reduced differential transform (15):

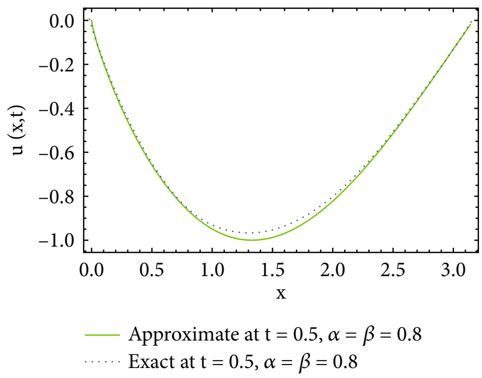
$$\begin{aligned}u(x,t)&=\sum_{k=0}^{\infty}U_k^{\alpha}(x)t^{k\alpha}=U_0^{\alpha}(x)+U_1^{\alpha}(x)t^{\alpha}+U_2^{\alpha}(x)t^{2\alpha}\\&\quad+U_3^{\alpha}(x)t^{3\alpha}+\cdots=\sin\left(\frac{x^{\beta}}{\beta}\right)-\frac{\sin\left(x^{\beta}/\beta\right)}{2!\alpha^2}t^{2\alpha}\\&\quad+\frac{\sin\left(x^{\beta}/\beta\right)}{4!\alpha^4}t^{4\alpha}-\frac{\sin\left(x^{\beta}/\beta\right)}{6!\alpha^6}t^{6\alpha}+\cdots\\&=\sin\left(\frac{x^{\beta}}{\beta}\right)\left(1-\frac{t^{2\alpha}}{2!\alpha^2}+\frac{t^{4\alpha}}{4!\alpha^4}-\frac{t^{6\alpha}}{6!\alpha^6}+\cdots\right)\\&=\sin\left(\frac{x^{\beta}}{\beta}\right)\sum_{k=0}^{\infty}\frac{(-1)^k(t^{\alpha})^{2k}}{(2k)!\alpha^{2k}}=\sin\left(\frac{x^{\beta}}{\beta}\right)\cos\left(\frac{t^{\alpha}}{\alpha}\right).\end{aligned} \tag{69}$$

Remark 20. When $\alpha=\beta=1$, equation (62) reduces to the classical (or nonfractional) space-time telegraph equation, and its exact solution is given by $u(x,t)=\sin x\cos t$.

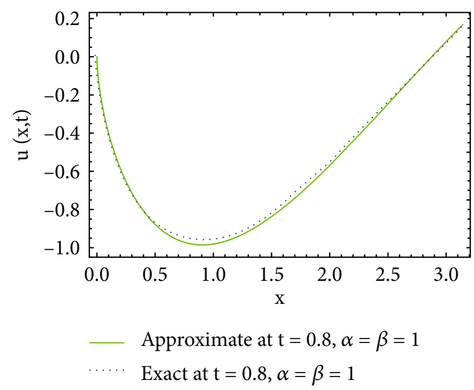
5.2. Numerical Simulation to Illustrative Examples. In this subsection, the numerical simulation of the considered problems was depicted using tables and figures.

6. Results and Discussion

This segment discusses the proposed method's precision and applicability by comparing the approximate and exact solutions using graphs and tables. Figures 1, 2, 3, and 4 depict the 3D plot eighth-order approximate solutions of Examples 1–4 obtained by the present method in comparison with the exact solutions at $\alpha=\beta=0.4$, 0.8, and 1. Figures 5, 6, 7, and 8 present the comparison of line plots of eighth-order approximate solutions from the proposed method and exact solutions of Examples 1–4 for different values of time variable t and fractional-order values α and β. As we can see from the figures, the line plots of the approximated series solution by RDTM almost



(a)



(b)

Figure 8: 2D solution plots of Example 4 obtained by the present method in comparison with the exact solutions for different values of fractional orders α and β.

coincide with those of the exact one. Moreover, the solution obtained by the present method becomes close to the exact solution as the number of order of approximation is increased and when the fractional values $\alpha, \beta \longrightarrow 1$. A comparative study between the exact and approximate solutions of each example in terms of absolute error at $x = 1$, and at $\alpha = \beta = 0.5$ and 1 for different values of time variable t are provided in Tables 1–4. As observed from the tables, the proposed method gives a small error near $t = 0$, but the error increases as $|t|$ increases. This is means that a better approximation can be achieved for small values of time t whatever the values of x is within the domain of interest.

7. Conclusion

In this paper, our main purpose was to inspect the competence of the RDTM as a valid technique for solving linear and nonlinear time-fractional one-dimensional conformable fractional telegraph equations with the initial value. The basic definitions and important results of both conformable fractional derivative and RDTM in the conformable fractional derivative sense are briefly discussed. After dealing with four distinct conformable fractional telegraph equations from mathematical physics problems, the outcomes of exemplary research by the proposed method gave us a series solution that converged very quickly to the exact solutions that we anticipated from previous numerical instances, as illustrated throughout the references [47, 49, 50, 59, 60]. Tables 1–3 and Figures 1, 2, 3, 4, 5, 6, 7, and 8 for each investigated example exhibit the eighth-term approximate solutions by the proposed method in comparison to exact solutions. As it is observed from the tables and figures, the approximated series solutions are almost the same as the exact solutions, and the error is very small, particularly as the values of $|t|$ approach to 0. Therefore, it is remarkable to observe that the RDTM is a simple, reliable, and powerful tool for solving such kinds of CFDEs, especially when the exact solution of such problems can be written as a closed form of a summation. Not only does it give solutions that agree perfectly with the exact solutions, but it does so with a minimal amount of computational work, saving us a lot of time and effort. At the last, by considering all above discussed facts and observing the assessment of RDTM, we can strongly recommend to the scientific community that the proposed method RDTM is an efficient mathematical tool used to obtain the general solution of complicated multidimensional differential equations that might discover the broader new applications in the scientific and technological field. Moreover, the author's future scope is to find the solution to higher-order nonlinear PDEs by using the RDTM with different noninteger fractional derivatives and make comparisons with other known numericals to confirm the effectiveness of the method.

Data Availability

The data statement of this manuscript is prepared according to the research data policy of the journal "Advances in Mathematical Physics."

Conflicts of Interest

The author declares that there is no conflict of interest.

Acknowledgments

The author wishes to express his gratitude to Mizan Tepi University's College of Natural Sciences and Mathematics Department for providing the required resources for this study.

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